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Edited by G. V. E. THOMPSON

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EDITORIALS

The Society's Publications

The postal ballot held recently showed an overwhelming majority in favour of an increase in the annual subscription rates (a report on the ballot and on the subsequent Extraordinary General Meeting will appear in a later issue of the *Journal*).

The extra income derived from the increased subscriptions will be used mainly to meet rises in postage and other expenses, but has also enabled Council to approve a further increase in the publications provided to members. After an interlude of one year, during which it appeared as a quarterly, the *Journal* now reverts to its normal frequency of publication—six issues per year. Each issue will continue to contain not less than 64 pages. This policy is necessary because of the greater volume of good-quality papers being submitted for publication. Thus, among the papers awaiting publication are several dealing with the present satellites, and it is imperative that they should appear as soon as possible (arrangements have been made to provide members with preprints in one case—see p. 311).

A further considerable increase in size and frequency would be desirable but is financially impracticable at present unless the revenue from advertisements can be improved. However, Council has approved a change in the format of the *Journal* (page size to be similar to that of *Spaceflight*), and this will of necessity include a slight increase in the bulk of each issue. This change will take place at the beginning of Volume 17 (in February, 1959).

As readers will note, apart from such regular features as TECHNICAL REVIEW, ABSTRACTS, etc., the present issue of the *Journal* contains but a single paper, mathematical in character. No apology is made for this, although it is realized that even some of our more highly-qualified Fellows may find it uninteresting or heavy going. The prime function of the *Journal* is the advancement of astronautics, and by enabling the Society to publish such material as this each member is playing his part in furthering this aim. The present paper is of particular interest in that it is a translation from the Russian; in view of Russian astronautical achievements we can perform a very useful task in making their underlying theoretical work more readily available to Western

mathematicians and we are grateful to *Engineering* for having provided this translation. We shall publish other translations of Russian papers in due course.

In addition to the changes in *Journal* publication policy announced above, it is hoped that it may be possible to issue *Spaceflight* six times a year as from January, 1959, so that we shall then be issuing twelve publications a year (*Spaceflight* and the *Journal* in alternate months). For this to be possible, it is essential that we receive greater advertising support, and members are asked to do all they can to obtain this.

Mention must also be made of some other publications of the Society. The Proceedings of the High Altitude and Satellite Rocket Symposium held at Cranfield last July will be available in about two months' time, and members will have the opportunity of purchasing copies at a special reduced price. Most of the text for a "B.I.S. Handbook" has now been written and this should be available next year. Another book—"Twenty-five Years of British Astronautics"—will be published this autumn in connection with the Society's Twenty-fifth Anniversary Celebrations.

Increasing Tempo of B.I.S. Activities

Plans for the Twenty-fifth Anniversary Celebrations in October are now well under way (but suggestions from members are still welcome). The culminating item will be a banquet, but various other activities are being organized, both in London and in the Branches. These will all be announced in the *Journal* at a later date.

Other symposia along the lines of that held at Cranfield last year are in course of preparation and will be announced to members as soon as definite arrangements for halls and accommodation have been made. In the last issue of the *Journal* mention was made of the forthcoming Amsterdam Congress of the I.A.F. Although the detailed programme will not be available for some time, more information about the arrangements has now been issued, and it is hoped that a large B.I.S. delegation will be attending the Congress (see p. 316).

These are only some of the activities which are evidence of the increasing tempo of the work of the Society. In addition, there is the Moonwatch programme, and the visits to research establishments, missile factories, etc., which are being arranged in the London area (see p. 310–1). Members who wish to take part in either Moonwatch or the visits should notify the Secretary as soon as possible.

Explorer

As this issue goes to press we learn that the first U.S. satellite is now in orbit. Congratulations to our Honorary Fellow, Dr. Wernher von Braun, and to all his team! We hope they will go on to even greater achievements.

A photograph of the launching is reproduced on p. 315. Other photographs and information concerning Explorer will appear in future issues of the Society's publications.

SOME VARIATION PROBLEMS CONNECTED WITH THE LAUNCHING OF ARTIFICAL SATELLITES OF THE EARTH*

By D. E. Okhotsimskii and T. M. Eneev

SUMMARY

This paper deals with the problem of placing an artificial satellite of the Earth in orbit. It is assumed that this is carried out with the help of a rocket accelerator consisting of one or several stages. The law of change in thrust with respect to time is investigated, with the object of delivering the satellite into its orbit with the minimum propellent consumption. The most economic regime of propellent consumption is sought.

The solution of these problems, carried out in a series of simplifying assumptions, allows one to form a definite concept of the characteristics of an optimum placing in orbit and also shows the way in which a vehicle can be made with the minimum initial weight.

In the first part we deal with the problem of choosing simultaneously the thrust control programme and the fuel consumption regime. In the second part we deal with the question of choosing an optimum programme for a multi-stage accelerator with a variable number of stages assuming that the propellent consumption regime is given. In the third part is a generalisation of the problem of placing in orbit in a central gravitational field, making allowance for the Earth's rotation.

1. Choice of optimum propellent consumption regime and optimum thrust control programme

In working out this problem we will neglect aerodynamic forces and assume that the terrestrial gravitational field is plane-parallel. The first hypothesis is justified by the fact that in establishing a satellite in orbit a considerable portion of the delivery trajectory will lie in the higher layers of the atmosphere where aerodynamic forces are insignificant. Replacement of the central gravitational field of the Earth by a plane-parallel field will be possible if we assume that the extent of the delivery trajectory is small compared with the radius of the Earth.

Both these assumptions are actually only approximately fulfilled. However, the function of the solution in its approximate form is of great interest because it allows us to obtain quite simply a full solution of the problem, to analyse the results obtained and to understand the fundamental mechanism of the phenomena.

The equations of motion of a satellite and vehicle projected on Cartesian coordinates can be written in the form (Fig. 1):

*Paper first published (*in Russian*) in *Uspekhi Fizicheskikh Nauk*, September, 1957, **63**, Supplement 1a, 5–32. An almost identical paper with the title "On the Establishment of an Artificial Satellite of the Earth in Orbit," by the same authors, was presented to the Eighth International Astronautical Congress, Barcelona, 6–12 October, 1957, as a communication from the Interdepartmental Commission on Interplanetary Travel of the Astronomical Council of the Academy of Sciences of the U.S.S.R., and will appear in the Proceedings of the Congress (to be published by Springer Verlag, Vienna). This translation is one provided by *Engineering*, and revised by G. V. E. Thompson.

$$\left.\begin{aligned} \frac{du}{dt} &= p\cos\phi \\ \frac{dw}{dt} &= p\sin\phi - g \\ \frac{dy}{dt} &= w \\ \frac{dx}{dt} &= u \end{aligned}\right\} \quad .. \quad .. \quad .. \quad (1.1)$$

where u and w are the horizontal and vertical projections of the velocity, p is the magnitude of the acceleration due to the reaction force, and ϕ is the inclination of the thrust to horizontal. As the thrust can be assumed to be directed along the longitudinal axis of the vehicle, ϕ can be taken as the inclination of the axis of the vehicle to the horizontal (i.e. the angle of pitch). In equations (1.1), x and y are the horizontal and vertical coordinates.

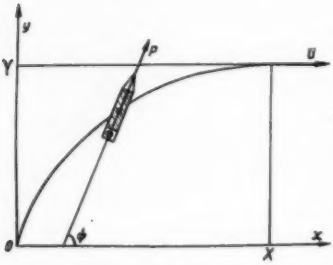


FIG. 1.

The latter equation of system (1.1) is only for determining the x coordinate. If we make no limitations as to distance of flight, then in solving the variational problem this equation can be discarded, inasmuch as coordinate x does not enter the other equations.

Let us discuss the velocity V which the satellite will have gained in the ideal case, when no forces act on it other than the thrust. We then have:

$$V = \int_0 p\,dt,$$

from whence:

$$p = \frac{dV}{dt}$$

Putting this into system (1.1) we can write the first three equations in the form:

$$\left.\begin{aligned} \frac{du}{dt} &= \frac{dV}{dt}\cos\phi \\ \frac{dw}{dt} &= \frac{dV}{dt}\sin\phi - g \\ \frac{dy}{dt} &= w \end{aligned}\right\} \quad .. \quad .. \quad .. \quad (1.2)$$

The value of V represents the available velocity which can be imparted to the satellite in the process of accelerating and placing in orbit.

The right-hand sides of the equations of motion (1.2) contain two indeterminate functions of time, $V(t)$ and $\phi(t)$. Let us put the problem in the form of choosing these functions in such a way that at the end of the period of delivery to a given height the horizontal velocity is a maximum. This also involves obtaining the solution of gaining the greatest height for a given speed and also obtaining given height and velocity with minimum propellent consumption.

Let us now determine the boundary conditions. Assuming that at the start of motion at $t = 0$ we have some value of height y_0 and some values of horizontal and vertical components of velocity u_0 and w_0. Obviously velocity V at the start of motion is zero. At the end of motion at the moment $t = T$ the height should equal $y = Y$, and the velocity must be directed horizontally, *i.e.*, $w = 0$. The value V at the end of motion we equate to some fixed value V_k. It is thereby assumed that there is some reserve of ideal velocity which could be used in a more rational manner. If we assume that the exit velocity, or specific thrust, is independent of the propellent consumption per second, the magnitude of the total ideal velocity V_k will be determined by the ratio of initial and final masses of the vehicle and will not depend on the regime of fuel consumption. This indicates that assigning a definite value V_k at the end of motion corresponds to determining a definite relation between initial and final masses, and given the mass of the satellite it fixes a definite initial mass of the vehicle. Thus the boundary conditions have the following form:

$$\left.\begin{aligned} &\text{for } t = 0\text{: } u = u_0, w = w_0, y = y_0, V = 0. \\ &\text{for } t = T\text{: } \qquad w = 0, y = Y, V = V_k. \end{aligned}\right\} \quad .. \quad .. \quad .. \quad (1.3)$$

The time of motion T over a portion of the path can either be given or can be chosen from the conditions of obtaining maximum velocity at the end of the motion. It is possible here to assume that the time of motion T over the portion of the flight path does not necessarily correspond with the time T_* that the jet is working, but need only satisfy the condition

$$T \geqslant T_*$$

where T_* is the given magnitude.

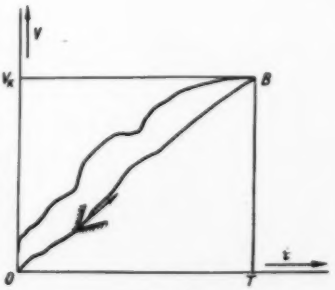


FIG. 2.

We will not impose any boundary conditions on function $\phi(t)$, except for the condition of continuity. Function $V(t)$ as an integral of non-negative quantities should be considered non-diminishing. A representative graph showing the dependence of V upon time is shown in Fig. 2. The horizontal portions correspond to motion without the engine working. The vertical parts correspond to instantaneous burning up of a part of the propellent. If we do not impose any limitations on the rate of consumption the allowable

line in plane (t, V) will be any line joining O and B along which V does not decrease. If we impose a limitation on the consumption rate, namely, that the line should satisfy a more rigid condition, then each element of curve emanating from a given point must lie within or on the boundaries of some angle, the upper side of which corresponds with motion with the greatest and the lower one with the least consumption rate. In the limiting case when the greatest allowable consumption is infinitely large (instantaneous burning) and the least is zero (motion with the engine switched off) the upper side of the angle will be vertical and the lower one horizontal.

The velocity at the end of any region of acceleration can be obtained by integrating the first equation of (1.2) within the range 0 to T:

$$U = \int_0^T \frac{dV}{dt} \cos \phi dt + u_0 \quad .. \quad .. \quad .. \quad (1.4)$$

In addition, we have two other equations which are also differentially connected. Let us re-write them in the form

$$\frac{dw}{dt} - \frac{dV}{dt} \sin \phi + g = 0 \quad .. \quad .. \quad .. \quad (1.5)$$

$$\frac{dy}{dt} - w = 0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (1.6)$$

We have also boundary conditions (1.3). Magnitudes u_0 and w_0 can be in the form

$$u_0 = v_0 \cos \theta_0; \; w_0 = v_0 \sin \theta_0 \quad .. \quad .. \quad (1.7)$$

Assuming V_0 given and varying angle θ_0 we get from the solution the value of the optimum angle of inclination of the vector of the initial velocity.

Now let us establish the auxiliary function:

$$I = v_0 \cos \theta_0 + \int_0^T \left[\frac{dV}{dt} \cos \phi + \lambda_1 \left(\frac{dw}{dt} - \frac{dV}{dt} \sin \phi + g \right) + \lambda_2 \left(\frac{dy}{dt} - w \right) \right] dt \; .. \; (1.8)$$

where λ_1 and λ_2 are as yet undetermined functions of time. Let us work out the variation of this function, varying both the initial angle ϕ_0 and the time of motion on the portion of the path T.

Carrying out the variation with respect to all the functions and variable parameters involved, and making use of the boundary conditions we obtain the following expression for the variation:

$$\delta I = - v_0 (\sin \theta_0 + \lambda_1 \cos \theta_0) \delta \theta + g \lambda_1 \delta T + \int_0^T \left\{ \left(-\sin \phi - \lambda_1 \cos \phi \right) \frac{dV}{dt} \delta \phi + \right.$$

$$\left. + \left[- \frac{d}{dt} \left(\cos \phi - \lambda_1 \sin \phi \right) \right] \delta V - \frac{d\lambda_2}{dt} \delta y - \left(\frac{d\lambda_1}{dt} + \lambda_2 \right) \delta w \right\} dt \quad .. \quad (1.9)$$

If the initial angle is not fixed, but is chosen from the optimum condition we can connect the initial angle θ_0 and the value of function $\lambda_1(t)$ at the beginning of motion:

$$\tan\theta_0 = -(\lambda_1)_{t=0} \quad .. \quad .. \quad .. \quad (1.10)$$

which is obtained by equating the first of the expressions in front of the integral sign to zero. If we equate the second expression to zero we get conditions at the end of an acceleration path:

$$(\lambda_1)_{t=T} = 0 \quad .. \quad .. \quad .. \quad .. \quad (1.11)$$

If the time of motion is fixed, then $\delta T = 0$ and the given condition for function $\lambda_1(t)$ no longer applies.

Now let us determine λ_1 and λ_2 in such a manner that the multipliers of δy and δw within the integral equal zero, and in this manner these variations are dropped from the expression for the variation of the function. If this is done the expression for the variation takes the form:

$$\delta I = \int_0^T \left\{ \left(-\sin\phi - \lambda_1\cos\phi \right) \frac{dV}{dt}\delta\phi + \left[-\frac{d}{dt}\left(\cos\phi - \lambda_1\sin\phi \right) \right] \delta V \right\} dt \; .. \; (1.12)$$

where function λ_1 must be determined from the equation

$$\frac{d\lambda_1}{dt} = -\lambda_2; \quad \frac{d\lambda_2}{dt} = 0 \quad .. \quad .. \quad .. \quad (1.13)$$

with boundary condition (1.11), which should be taken into consideration in the case when T is not fixed.

From the second equation of (1.13) we have:

$$\lambda_2 = \text{const.} \quad .. \quad .. \quad .. \quad .. \quad (1.14)$$

Putting this value into the first equation of (1.13) we obtain:

$$\lambda_1 = C_1 + C_2 t \quad .. \quad .. \quad .. \quad .. \quad (1.15)$$

where C_1 and C_2 are some constants. In the case when T is not fixed we have the relation

$$C_1 + C_2 T = 0$$

and the expression for λ_1 transforms into:

$$\lambda_1 = -C_2(T - t) \quad .. \quad .. \quad .. \quad (1.16)$$

Formulae (1.15) and (1.16) demonstrate that the value of λ_1 is a linear function of time. Note that these formulae contain the same number of arbitrary constants because in formula (1.16) instead of C_1 we introduce the unknown magnitude T.

If we equate to zero the expression for variation of $\delta\phi$ within the integral in formula (1.12) we obtain:

$$\tan\phi = -\lambda_1 \quad .. \quad .. \quad .. \quad (1.17)$$

and using formula (1.15) we obtain an expression for the optimum programme in pitch, in the form:

$$\tan\phi = -(C_1 + C_2 t) \quad .. \quad .. \quad .. \quad (1.18)$$

If T is not fixed, then in accordance with (1.16) we have

$$\tan \phi = C_2(T - t) \qquad .. \quad .. \quad .. \qquad (1.19)$$

Thus, if the time is not fixed then the path angle must be zero at the end of motion, *i.e.*, the axis of the vehicle must be horizontal.

The formula for the optimum programme can be re-written in the form:

$$\tan \phi = \tan \phi_0 - C_2 t \qquad .. \quad .. \quad .. \qquad (1.20)$$

where ϕ_0 is the path angle at the initial instant.

In this way we arrive at the conclusion that in the optimum programme the tangent of the pitch angle must be a linear function of time. The two parameters entering into the general expression for the programme should be so chosen that they satisfy the boundary conditions.

Combining formulae (1.10) and (1.17) shows that the optimum initial angle v_0 should satisfy the condition:

$$\tan \theta_0 = \tan \phi_0 \qquad .. \quad .. \quad .. \qquad (1.21)$$

This shows that the direction of the longitudinal axis of the vehicle at the initial instant must coincide with the direction of the initial velocity. In the case where angle θ_0 is already given this condition may not be fulfilled.

Formulae (1.18)–(1.20) give the law of change of pitch angle for any relation $V(t)$. We will now choose function $V(t)$ for the best manner, assuming that for each such function we always choose the best law of change of pitch angle.

Then the variation of the function takes the form:

$$\delta I = \int_0^T \left\{ -\left[\frac{d}{dt}\left(\sin \phi - \lambda_1 \cos \phi \right) \right] \delta V \right\} dt \qquad .. \qquad (1.22)$$

If we fulfil the differential conditions (1.5) and (1.6), functions (1.4) and (1.8) coincide. If these conditions are identically fulfilled during variation, the variation of the functions must then also coincide. Therefore formula (1.22) allows us to estimate the variation in velocity δU at the end of a region of acceleration for a change in regime of propellent consumption and the function $V(t)$ connected with it. It is essential to note that the expression for variation (1.22) will only be true if all changes in V come to an end before instant T.

Formula (1.22) can be transformed into:

$$\delta U = \int_0^T \Phi(t).\delta V.dt \qquad .. \quad .. \quad .. \quad .. \qquad (1.23)$$

where

$$\Phi = \frac{C_2(\tan \phi_0 - C_2 t)}{\sqrt{(\tan \phi_0 - C_2 t)^2 + 1}} \qquad .. \quad .. \quad .. \qquad (1.24)$$

or

$$\Phi = C_2 \sin \phi \qquad .. \quad .. \quad .. \quad .. \quad .. \qquad (1.24a)$$

Formulae (1.23) and 1.24) show that function $\Phi(t)$ within the integral factor for variation in δV, depends only on time. Over the path $(0, T)$ the function can either have the same sign all the time or it can change in sign, for

formula (1.24) shows that change in sign of Φ is connected with the change in sign of the numerator and can therefore only take place once.

From the foregoing features of function $\Phi(t)$ it is easy to show that the "extreme" of the function is attained in functions $V(t)$ by discontinuous change. Now let us take a look at various possible cases:

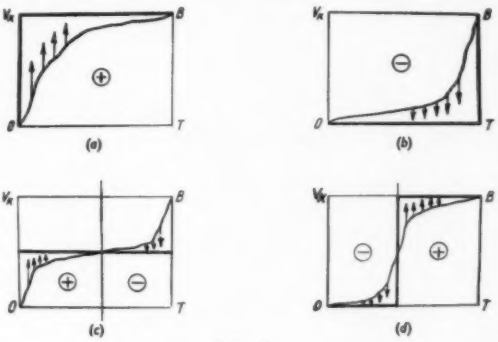


FIG. 3.

(*A*). *Expression* Φ *is always positive* (Fig. 3*a*). In this case the burning of the whole reserve of propellent should take place instantaneously at the very start of motion because any other possible line in the plane (t, V) can, in the worst case, be varied upwards, whereby we get $\delta U > 0$, which signifies an increase in velocity at the end of the path.

(*B*). *The expression* Φ *is always negative* (Fig. 3*b*). As above, we see that in this case the best way of using propellent is the instantaneous consumption of the whole propellent supply at the very end of the path at $t = T$.

(*C*). *Change in sign of* Φ *proceeds from plus to minus* (Fig. 3*c*). In this case the "extreme" can only be attained on broken sections consisting of vertical, horizontal and vertical ones. Any other allowable line can be varied in each region of constant sign in such a way that the increase in final velocity will be positive. So as to determine the position of the horizontal section we write the expression for variation of final velocity for a vertical displacement of the section:

$$\delta U = \delta V \int_0^T \Phi(t).dt \quad .. \quad .. \quad .. \quad .. \quad (1.25)$$

If the value of the integral is positive, then this case is analogous to Case *A*; if negative, then to Case *B*. If the value of the integral equals zero, it means that the "extreme" function can be represented by any intermediate position of the horizontal section. Thus for given conditions it can be advantageous to burn some of the propellent instantaneously right at the start of the motion and the remaining part instantaneously at the very end of the movement.

(*D*). *Change of sign of* Φ *goes from minus to plus* (Fig. 3*d*). In this case expenditure of the whole propellent reserve should be done instantaneously at some instant of motion.

All the above possibilities can be realised under given conditions.

Now let us deal with the case where the time of motion is not fixed but is chosen from the conditions of maximum final velocity. The final angle is also

not fixed but is chosen as optimum. In this case we have according to (1.11), (1.17) and (1.20):

$$\tan \phi_k = \tan \phi_0 - C_2 T = 0 \quad .. \quad .. \quad (1.26)$$

from whence:

$$C_2 = \frac{\tan \phi_0}{T} \quad .. \quad .. \quad .. \quad .. \quad (1.27)$$

In this case the numerator in formula (1.24) for Φ will take the form

$$\tan^2 \phi_0 \left(1 - \frac{t}{T}\right) \quad .. \quad .. \quad .. \quad .. \quad (1.28)$$

and will be positive over the whole range $(0, T)$. This means that Case A will be realised. We can see that the greatest value of final velocity U occurs when the initial angle and time of motion are chosen from the optimum conditions and is obtained by instantaneous burning of all the propellent reserve at the start. The directions of the initial velocity v_0 and supplementary velocity V must be equal. We will omit suffix k from V from now on.

We will now assume that initial velocity v_0 is included in V, whereupon division of the total available velocity into two parts, if necessary, is carried out by optimising. Obviously we can limit this investigation by the case of $v_0 = 0$. Now let us study this case both for free, and also for fixed, time of motion over the path. We will ascertain under which conditions of the parameters a solution is possible and we will study the relation between the character of the optimum motion and the magnitude of the time of establishment T.

It is clear that if $v_0 = 0$ only cases A and C are possible. To satisfy the boundary conditions we have relations:

$$V_1 \sin \phi_1 + V_2 \sin \phi_2 - gT = 0 \quad .. \quad .. \quad (1{\cdot}29)$$

$$V_1 \sin \phi_1 . T - \tfrac{1}{2} g T^2 = Y \quad .. \quad .. \quad .. \quad (1.30)$$

where Y is given height, V_1 is the velocity obtained at the initial instant of motion and V_2 the velocity obtained at the final moment of motion. The sum

$$V_1 + V_2 = V \quad .. \quad .. \quad .. \quad (1.31)$$

where V is the available reserve of velocity.

If T is not fixed, V_2 equals 0. Equations (1.29) and (1.30) give a relation

$$\tfrac{1}{2} g T^2 = Y \quad .. \quad .. \quad .. \quad .. \quad (1.32)$$

which can be used for determining time T:

$$T = \sqrt{2Y/g} \quad .. \quad .. \quad .. \quad .. \quad (1.33)$$

Putting this into either of the equations (1.29) or (1.30), we get

$$\sin \phi_1 = \sqrt{2gY}/V \quad .. \quad .. \quad .. \quad (1.34)$$

The problem can be solved if the right-hand side is less than unity. This means that for a given available velocity V the height of delivery Y must not be too great, and conversely for a given height the reserve of available velocity must be sufficiently great—in each case it must be greater than that velocity

which is necessary for projecting a body vertically upwards to a height Y. If we fulfil this condition, the horizontal velocity at the end of delivery path will be:

$$U = V.\cos\phi_1 \quad .. \quad .. \quad .. \quad .. \quad (1.35)$$

or

$$U = V\sqrt{1 - (2gY/V^2)} \quad .. \quad .. \quad (1.36)$$

If T is fixed, then in general condition (1.32) is not fulfilled and cannot be found by solving at $V_2 = 0$. The solution should be found in scheme C for an intermediate position of the horizontal section.

It is easy to see that the expression for function Φ can be put in the form:

$$\Phi = \frac{d}{dt}\left(\frac{1}{\cos\phi}\right) \quad .. \quad .. \quad .. \quad .. \quad (1.37)$$

Therefore, equating to zero the integral in formula (1.25) gives:

$$\cos\phi_1 = \cos\phi_2 \quad .. \quad .. \quad .. \quad (1.38)$$

from whence

$$\phi_2 = \pm\phi_1 \quad .. \quad .. \quad .. \quad .. \quad (1.39)$$

We will look at both possibilities and we will ascertain for which relations between the parameters they will be realised:

1. Let $\phi_2 = \phi_1$. From formula (1.29) we have:

$$\sin\phi_1 = gT/V \quad .. \quad .. \quad .. \quad (1.40)$$

and by using (1.30) and (1.31) we obtain:

$$V_1 = \tfrac{1}{2}V[1 + (2Y/gT^2)] \quad .. \quad .. \quad .. \quad (1.41)$$

$$V_2 = \tfrac{1}{2}V[1 - (2Y/gT^2)] \quad .. \quad .. \quad .. \quad (1.42)$$

As V_2 must be positive, we see that the given case takes place if the given time of motion over the delivery path satisfies the condition

$$T > \sqrt{2Y/g} \quad .. \quad .. \quad .. \quad (1.43)$$

i.e., is greater than the optimum time of delivery. For final velocity we have the formula:

$$U = V\sqrt{1 - (gT/V)^2} \quad .. \quad .. \quad .. \quad (1.44)$$

The condition that the expression gT/V must be less than unity is a requirement imposed on the magnitude of the available reserve of velocity.

2. Let $\phi_2 = -\phi_1$. Then we have:

$$\sin\phi_1 = 2Y/VT \quad .. \quad .. \quad .. \quad (1.45)$$

For velocities we obtain:

$$V_1 = \tfrac{1}{2}V[1 + (gT^2/2Y)] \quad .. \quad .. \quad .. \quad (1.46)$$

$$V_2 = \tfrac{1}{2}V[1 - (gT^2/2Y)] \quad .. \quad .. \quad .. \quad (1.47)$$

This solution is valid if the given time T is less than the optimum, *i.e.*, if:

$$T < \sqrt{2Y/g} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (1.48)$$

The final velocity is obtained from the formula:

$$U = V\sqrt{1 - (2Y/VT)^2} \quad .. \quad .. \quad .. \quad (1.49)$$

The condition that the expression $2Y/VT$ be less than unity is analogous to the foregoing one and can be regarded as a condition that the available reserve of velocity V be sufficiently high.

These formulae show that the magnitude of the supplementary velocity V_2 imparted at the end of motion on the delivery path depends on how much a given time of delivery differs from the optimum and will be the greater, the greater this difference. If the given time is equal to the optimum, then $V_2 = 0$ and both cases reduce to the previous one. We note that in both cases:

$$V_2 < V_1$$

i.e., the second supplementary velocity is always less than the first.

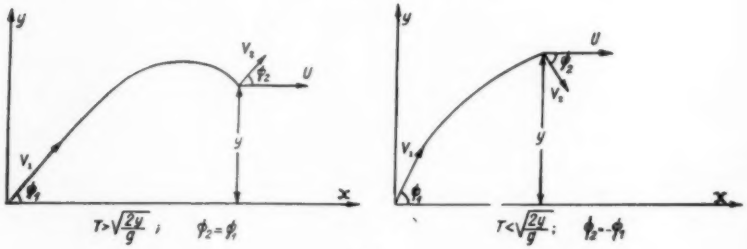


FIG. 4.

Conditions (1.43) and (1.48) mean that the increment in velocity must take place in the first case on the descending, and in the second case on the ascending branch of the parabola obtained as a result of motion with initial velocity V_1. Both cases are shown in Fig. 4. In the first case, when the given time is greater than the optimum, the increase in velocity V_2 takes place on the descending branch of the parabola and is directed parallel to the initial velocity V_1. In the second case, when the given time of delivery is less than the optimum, the increment in velocity V_2 takes place on the rising branch of the parabola and is directed downwards at an angle to the horizontal equal in magnitude to the initial angle ϕ_1.

Over the delivery path the problems of rising to a given height and imparting a given velocity in a horizontal direction must be solved. The results obtained show that in the most favourable case, when the delivery time is also chosen as optimum, the increase in velocity and the increase in height should be carried out by applying an instantaneous impulse at the very beginning of motion. Delivery into a horizontal direction of motion must be done by gravitation. If the given time of delivery is different from the optimum, then for delivery into a horizontal direction it is necessary to employ an impulse imparted at the end of the path. If this given excess time is too great, the deviation in velocity vector due to gravity is greater than necessary and the vertical component of

a second impulse must be fired upwards. If the given time is small, gravity will not have time to influence the velocity vector and the missing rotation must be imparted by a second impulse having a vertical component directed downwards.

In obtaining the solutions we started from the assumption of the possibility of instantaneously applying impulse and instantaneous burning of the whole propellent supply or part of it. As actually this is impossible, the best solution of the stated problem of establishment is obtained by replacing instantaneous burning by burning the propellent at the maximum specific rate, and an interval between the application of impulses is motion with minimum specific fuel consumption, or, even better, motion with the engine switched off.

It was deduced above that in motion in a plane-parallel gravitational field the best path is obtained by applying a single impulse at the beginning of motion. This solution is inapplicable for the case of placing into an orbit with a central field of force, because it is impossible to obtain, for instance, a circular orbit or an orbit which does not intersect or is not tangential to the surface of the Earth. Therefore, for application of the results obtained above to the solution of the problem concerning establishment in a central force field, the most important factor is the problem of delivering into the orbit in a given time. If the given time is not too great, the dimensions of the delivery trajectory will not be great with respect to the radius of the Earth and the hypotheses on which the approximate study was based will not be significantly invalidated.

Let us work out an example which enables us to estimate the extent of the delivery trajectory and the amount of velocity reserve available for various values of given time of delivery. In this example we will limit ourselves to the most important case when the given time of delivery does not exceed the optimum time in a plane-parallel force field.

Suppose it is necessary to deliver a satellite to a height $Y = 300$ km. with a horizontal velocity $U = 7{,}900$ m./sec.

According to the formula:

$$T = \sqrt{2Y/g}$$

we obtain the optimum delivery time in a plane-parallel force field:

$$T_{\text{opt.}} = 248 \text{ sec.}$$

In Fig. 5 is given for the region $0{\cdot}4\ T_{\text{opt.}} < T < T_{\text{opt.}}$ a graph of the relation between T and the magnitude of the total usable available velocity V, the initial impulse v_1, the initial angle ϕ_1, and the horizontal range X. From the diagram it is evident that with a reduction in the delivery time the total usable velocity increases, the magnitude of the initial impulse falls and the final impulse increases. The magnitude of the initial angle increases with decrease in delivery time, the trajectories become steeper and the horizontal range decreases. The values obtained for the horizontal range are very much less than the radius of the Earth. Therefore this example can be regarded as an approximate calculation for an optimum delivery of satellite into its orbit.

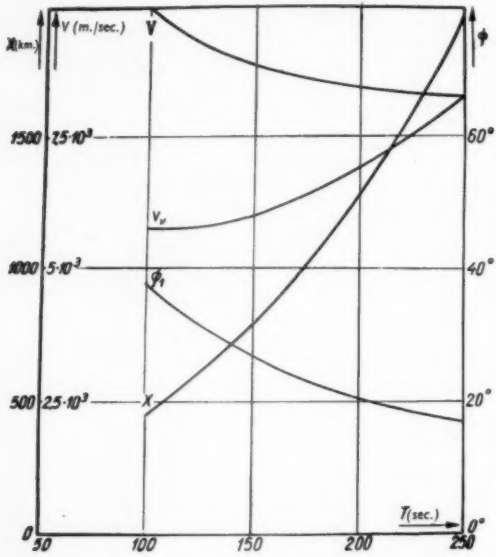


FIG. 5.

Finally, let us note at the end of this section, that, as was shown above, the linear law relating change in tangent of pitch angle with time is independent of change in function $V(t)$. It is applicable both for single and for multi-stage vehicles, and also for the case when there are intervals between the end of burning of one stage and the beginning of burning of the next. This result will be used in the following section in studying delivery using vehicles of the step-rocket type.

Let us also note that under different conditions of the variational problem the law for the tangent of the pitch angle can differ from the linear one. Thus, for example, if on selecting an optimum time of motion we fix the horizontal range of the delivery portion, the optimum law of change of tangent of pitch angle with respect to time will be fractionally linear. Below, in Section **3** we discuss results of solving the variational problems under various conditions, taking into consideration the earlier factors which we left out and we obtain the optimum laws of change in pitch angle with respect to time.

2. Motion under Conditions of a Given Propellent Consumption Regime

We have already found that the best pitch control law is that giving the linear dependence of the tangent of the pitch angle upon time:

$$\tan \phi = -(C_1 + C_2 t) \quad .. \quad .. \quad .. \quad (2.1)$$

Integrating the equations of motion (1.1)–(1.4) we obtain expressions for the projection velocity and for the co-ordinates at some instant of time t_1:

$$u = \int_0^{t_1} \hat{p}(t). \cos \phi . dt + u_0 \quad .. \quad .. \quad .. \quad .. \quad (2.2)$$

$$w = \int_0^{t_1} \hat{p}(t). \sin \phi . dt + w_0 - g t_1 \quad .. \quad .. \quad .. \quad (2.3)$$

$$x = \int_0^{t_1} (t_1 - t).p(t).\cos\phi.dt + u_0 t_1 + x_0 \quad .. \quad .. \quad (2.4)$$

$$y = \int_0^{t_1} (t_1 - t).p(t).\sin\phi.dt + w_0 t_1 + y_0 - \tfrac{1}{2} g t_1^2 \quad .. \quad (2.5)$$

Using (2.1), the quantities $\cos\phi$ and $\sin\phi$ can be expressed as functions of time:

$$\cos\phi = \frac{1}{\sqrt{1 + (C_1 + C_2 t)^2}} \quad .. \quad .. \quad .. \quad (2.6)$$

$$\sin\phi = -\frac{C_1 + C_2 t}{\sqrt{1 + (C_1 + C_2 t)^2}} \quad .. \quad .. \quad .. \quad (2.7)$$

As function $p(t)$ can be taken as given, the integrals in the right-hand sides of formulae (2.2)–(2.5) can generally be evaluated.

The right-hand sides of formulae (2.2)–2.5) contain two arbitrary constants, which are connected with the control programme. Selecting these constants, it is possible, in general, to secure that for a fixed time of movement over the acceleration portion, the given height y_k at the end of the acceleration portion will be attained and the vertical velocity component will be equal to zero. Having selected constants in this way, by using formulae (2.2) and (2.4) it is possible to determine the magnitude of the velocities u_k and x_k.

In order to be able to actually integrate the right-hand side of formulae (2.2)–(2.5), it is essential to know the relation between the acceleration due to reaction and the time, *i.e.*, to know function $p(t)$. Note that function $p(t)$ is in the general case discontinuous, for instance in the case of step-rockets. As step-rockets for the artificial satellite are of the greatest interest, we will make a more detailed study of the optimum motion of a step-rocket below.

For the case of motion of a step-rocket, using equations (2.2)–(2.5) the kinematic parameters at the end of the delivery trajectory can be represented as:

$$u_k = u_0 + \sum_{i=1}^{n} u_k^{(i)} \quad .. \quad .. \quad .. \quad .. \quad (2.8)$$

$$w_k = w_0 + \sum_{i=1}^{n} w_k^{(i)} \quad .. \quad .. \quad .. \quad .. \quad (2.9)$$

$$x_k = x_0 + \sum_{i=1}^{n} x_k^{(i)} \quad .. \quad .. \quad .. \quad .. \quad (2.10)$$

$$y_k = y_0 + \sum_{i=1}^{n} y_k^{(i)} \quad .. \quad .. \quad .. \quad .. \quad (2.11)$$

where:

$$u_k^{(i)} = \int_{t_0^{(i)}}^{t_k^{(i)}} p_i(t).\cos\phi.dt \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (2.12)$$

$$w_k^{(i)} = \int_{t_0^{(i)}}^{t_k^{(i)}} p_i(t).\sin\phi.dt - g\,(t_k^{(i)} - t_0^{(i)}) \quad .. \quad .. \quad .. \quad .. \quad .. \quad (2.13)$$

$$x_k^{(i)} = \int_{t_0^{(i)}}^{t_k^{(i)}} (t_k^{(i)} - t)p_i(t).\cos\phi.dt + (t_k^{(i)} - t_0^{(i)}).\sum_{q=0}^{i=1} u_k^{(q)} \quad .. \quad .. \quad .. \quad (2.14)$$

$$y_k^{(i)} = \int_{t_0^{(i)}}^{t_k^{(i)}} (t_k^{(i)} - t).p_i(t).\sin\phi.dt + (t_k^{(i)} - t_0^{(i)}).\sum_{q=0}^{i=1} w_0^{(q)} - \tfrac{1}{2}g(t_k^{(i)} - t_0^{(i)}) \quad .. \quad (2.15)$$

$$u_k^0 = u_0 \qquad w_k^0 = w_0$$

Here $t_0^{(i)}$ and $t_k^{(i)}$ represent the time at the initial and final moments of motion, $u_k^{(i)}$ and $w_k^{(i)}$ are the magnitudes of the increase in horizontal and vertical components of velocity, $x_k^{(i)}$ and $y_k^{(i)}$ are the increases in horizontal and vertical co-ordinates during the time of motion and $p_i(t)$ is the acceleration due to reactive force, where all the quantities relate to motion of the i^{th} stage.

It is clear that formulae (2.12)–(2.15) embrace also the case when there are pauses between the working of the engines in the different stages. In this case we can assume that the time interval corresponding to the pause is related to a given rocket stage for which:

$$p_i(t) \equiv 0$$

We will evaluate the integrals of the right-hand sides of formulae (2.12)–(2.15) for one very important particular case, in which the motion of each rocket stage takes place at constant thrust. In this case, for the i^{th} step:

$$p_i(t) = gP_i/G_i \quad .. \quad .. \quad .. \quad .. \quad (2.16)$$

where P_i is the thrust of the engine, G_i is the mass. The mass G_i can be represented in the form:

$$G_i = G_0^{(i)}.\mu^{(i)} = G_0^{(i)}\,[1 - (1 - \mu_k^{(i)}).(t - t_0^{(i)})/T_i] \quad .. \quad .. \quad (2.17)$$

where $G_0^{(i)}$ is the initial mass, $\mu_k^{(i)}$ is the final mass ratio, T_i is the time during which the engine is working. Introducing the relation:

$$v_0^{(i)} = G_0^{(i)}/P_i \quad .. \quad .. \quad .. \quad .. \quad (2.18)$$

we obtain an expression for $p_i(t)$:

$$p_i(t) = \frac{g}{v_0^{(i)}.[1 - (1 - \mu_k^{(i)})(t - t_0^{(i)})/T_i]} \quad .. \quad .. \quad (2.19)$$

Determining engine thrust by the formula:

$$P_i = -\frac{c_i}{g}.\frac{dG_i}{dt} \quad .. \quad .. \quad .. \quad .. \quad (2.20)$$

where c_i is the exit velocity of the gases from the engine nozzle, it is easy to obtain the following formula for time T_i:

$$T_i = (c_i/g).v_0^{(i)}.(1 - \mu_k^{(i)}) \quad .. \quad .. \quad (2.21)$$

Note that for a constant thrust engine the consumption per second is constant and the mass of the rocket is a linear function of time. For each stage it is therefore possible to assume that the tangent of the pitch angle is linearly dependent, not on time, but on the mass of the stage or on its mass ratio. We then have:

$$\tan \phi = A_i + B_i.\mu^{(i)} \quad .. \quad .. \quad .. \quad (2.22)$$

where A_i and B_i are certain constants. At the beginning of motion of the stage, $\mu^{(i)} = 1$ and

$$\tan \phi_0^{(i)} = A_i + B_i \quad .. \quad .. \quad .. \quad (2.23)$$

At the end of the operation of the engine of the step, $\mu^{(i)} = \mu_k^{(i)}$, and

$$\tan \phi_k^{(i)} = A_i + B_i.\mu_k^{(i)} \quad .. \quad .. \quad .. \quad (2.24)$$

Here $\phi_0^{(i)}$ and $\phi_k^{(i)}$ are the values of the pitch angle respectively at the beginning and end of the motion of the stage.

Putting the expression for $p_i(t)$ from (2.19) and the expressions for $\sin \phi$ and $\cos \phi$ from (2.6) and (2.7) into formulae (2.8)–(2.15), we obtain after integrating:

$$u_k = u_0 + \sum_{i=1}^{n} c_i.f_1^{(i)} \quad .. \quad .. \quad .. \quad (2.25)$$

$$w_k = w_0 + \sum_{i=1}^{n} c_i.[f_2^{(i)} - v_0^{(i)}.(1 - \mu_k^{(i)})] \quad .. \quad (2.26)$$

$$x_k = x_0 + \sum_{i=1}^{n} (c_i^2/g)v_0^{(i)}.[f_3^{(i)} + (1 - \mu_k^{(i)}).\sum_{q=0}^{i-1}(c_q/c_i).f_1^{(q)}] \quad .. \quad (2.27)$$

$$y_k = y_0 + \sum_{i=1}^{n} (c_i^2/g)v_0^{(i)} \left\{ f_4^{(i)} - \tfrac{1}{2}v_0^{(i)}(1 - \mu_k^{(i)}) + \right.$$

$$\left. + (1 - \mu_k^{(i)}).\sum_{q=0}^{i-1}(c_q/c_i).[f_2^{(q)} - v_0^{(q)}.(1 - \mu_k^{(q)})] \right\} \quad .. \quad (2.28)$$

where the non-dimensional functions $f_1^{(i)}$, $f_2^{(i)}$, $f_3^{(i)}$, and $f_4^{(i)}$ are determined by the expressions:

$$f_1^{(0)} = u_0/c_0, \qquad f_2^{(0)} = w_0/c_0$$

$$f_1^{(i)} = \frac{1}{\sqrt{1 + A_i^2}} \left\{ \text{arc sh} \left(A_i + \frac{1 + A_i^2}{\mu_k^{(i)}.B_i} \right) - \text{arc sh} \left(A_i + \frac{1 + A_i^2}{B_i} \right) \right\} \quad (2.29)$$

$$f_2^{(i)} = A_i f_1^{(i)} + \text{arc sh}\,(A_i + B_i) - \text{arc sh}\,(A_i + B_i\mu_k^{(i)}) \quad .. \quad (2.30)$$

$$f_3^{(i)} = (1/B_i)[\text{arc sh}\,(A_i + B_i) - \text{arc sh}\,(A_i + B_i\mu_k^{(i)})] - \mu_k^{(i)}.f_1^{(i)} \quad .. \quad (2.31)$$

$$f_4^{(i)} = (1/B_i)[\sqrt{1 + (A_i + B_i)^2} - \sqrt{1 + (A_i + B_i\mu_k^{(i)})^2}] - \mu_k^{(i)} f_2^{(i)} \quad .. \quad (2.32)$$

$$(i = 1, 2, 3, \ldots\ldots, n)$$

We notice in addition that $\nu_0^0 = 0$. Constants A_i and B_i entering formulae (2.29)–(2.32) because of the discontinuous control programme, and also formulae (2.22)–(2.24) are mutually connected by $2(n-1)$ relations.

$$A_i + B_i\mu_k^{(i)} = A_{i+1} + B_{i+1} \quad .. \quad .. \quad (2.33)$$

$$\frac{B_i}{B_{i+1}} = \frac{\nu_0^{(i)}}{\nu_0^{(i+1)}} \cdot \frac{c_i}{c_{i+1}} \quad .. \quad .. \quad .. \quad .. \quad (2.34)$$

$$i = 1, 2, \ldots, (n-1)$$

Relation (2.34) can be obtained in the following way. In accordance with (2.1), $\frac{d}{dt}(\tan\phi) = -C_2 = \text{const}$. On the other hand, in accordance with (2.17) and (2.22):

$$\frac{d}{dt}(\tan\phi) = \frac{d}{d\mu^{(i)}}(\tan\phi) \cdot \frac{d\mu^{(i)}}{dt} = -B_i \cdot \frac{1-\mu_k^{(i)}}{T_i} = -C_2 \quad .. \quad (2.35)$$

From (2.35), bearing in mind (2.21), it is easy to obtain (2.34).

Formulae (2.21)–(2.25) allow complete evaluation of experimental motions. So as to obtain motions conforming to given isoperimetric conditions, it is essential by suitable methods to choose two arbitrary constants in the control programme C_1 and C_2 or, what amounts to the same thing, any of two constants from the series of $2n$ quantities A_i and B_i, for example A_1 and B_1 or A_n and B_n. Equating w to zero, and y to the given quantity y_k, we obtain, for the determination of A_i and B_i, a system of two transcendental equations, whose solutions can be found only by trial and error. It is therefore simpler not to determine A_i and B_i, but to assume them, and from these to calculate all the other quantities. By changing $\mu_k^{(i)}$, A_i and B_i over some considered range, it is possible to obtain a motion with different values of $\nu_0^{(i)}$, y_k and u_k. In this way the calculations will be much more economical, because it is not necessary to solve equations and it is only necessary to calculate by formulae.

We will now study one very interesting particular case of a step-rocket, so as to illustrate the method described above.

We will discuss a step-rocket in which all n stages are similar in respect of their basic characteristics, *i.e.*, a rocket in which:

$$c_i = c;\ \mu_k^{(i)} = \mu_k;\ \nu_0^{(i)} = \nu_0 \quad .. \quad .. \quad (2.36)$$

We will take A_n and B_n as arbitrary constants for the control programme. All other constants A_i and B_i can be easily expressed through A_n and B_n. Actually, in conformity with (2.34):

$$B_i = B_n \quad .. \quad .. \quad .. \quad .. \quad (2.37)$$

and from (2.33), using also (2.37), we easily obtain:

$$A_i = A_n + B_n(1-\mu_k)(n-i) \qquad i = 1, 2, \ldots, n \quad .. \quad (2.38)$$

Now the way to calculate is as follows. Given μ_k, A_n and B_n, and using formula (2.38) we determine by formulae (2.29)–(2.32) quantities $f_1^{(i)}$, $f_2^{(i)}$,

$f_3^{(i)}$ and $f_4^{(i)}$. From the condition $w_k = 0$ at the end of motion we have the formula:

$$\nu_0 = \frac{\sum_{i=1}^{n} f_2^{(i)}}{n(1-\mu_k)} \qquad .. \quad .. \quad .. \quad (2.39)$$

We can use this formula for determining ν_0. After this we determine quantities x_k, y_k and u_k by formulae (2.25), (2.27) and (2.28).

In doing this kind of calculation it is possible to deal not necessarily with A_n and B_n but with some values which depend on them. For instance, we can be given the magnitude of the pitch angle at the end of the trajectory and value of the mass ratio, and we can calculate a series of trajectories, varying the magnitude of the pitch angle at the beginning of the acceleration. Denoting the initial pitch angle of the first stage $\phi_0^{(1)}$ as ϕ_0 we will have:

$$\tan \phi_0 = A_1 + B_1 \qquad .. \quad .. \quad .. \quad (2.40)$$

In accordance with (2.24) we will get

$$\tan \phi_k = A_n + B_n \mu_k \qquad .. \quad .. \quad .. \quad (2.41)$$

where ϕ_k denotes the final pitch angle for the last n^{th} step, $\phi_k^{(n)}$. Finally bearing in mind that according to formula (2.38):

$$A_1 = A_n + B_n(1-\mu_k)(n-1) \qquad .. \quad .. \quad (2.42)$$

and that $B_1 = B_n$ from relations (2.40), (2.41) and (2.42) we obtain formulae which connect A_n and B_n with ϕ_0 and ϕ_k:

$$B_n = \frac{\tan \phi_0 - \tan \phi_k}{n(1-\mu_k)}$$

$$A_n = \tan \phi_k - \mu_k . B$$

The results of calculations are shown in Fig. 6, 7 and 8. In these calculations were made for two-stage (Fig. 6), three-stage (Fig. 7) and four-stage rockets (Fig. 8). Abscissae of the graphs show the final rocket velocity u_k both in dimensional and non-dimensional form (see below); the ordinates give the final delivery height y_k. When carrying out the calculations, the values of the pitch angles at the end of the trajectory were taken as:

$$\phi_k = 0°;\ -10°;\ -20°;\ -30°;\ -40°;\ -50°;\ -60°.$$

Values of μ_k were taken as:

$$\mu_k = 0{\cdot}1;\ 0{\cdot}2;\ 0{\cdot}3;\ 0{\cdot}4;\ 0{\cdot}5$$

For these values of the parameters curves were constructed along which ϕ_0 was varied. The limiting right-hand point of each curve corresponds to $\phi_0 = 60°$. Exceptions are curves at which points corresponding to $\phi_0 = 60°$, lie under the abscissae axis; values of ϕ_0 for the limiting right-hand points on

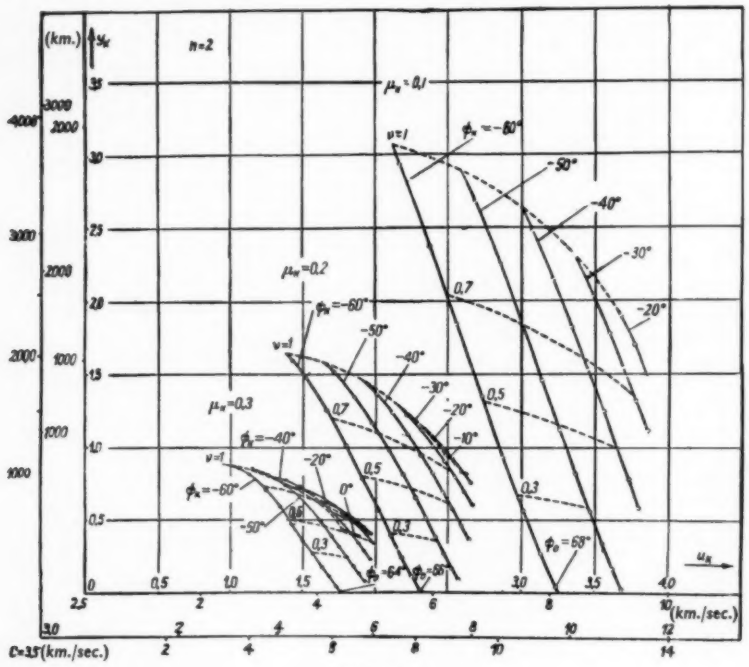


FIG. 6.

these curves are noted separately on the graphs. The remaining points ϕ_0 denoted on the graphs by circles are taken over the intervals $\Delta\phi_0 = 2°$ in the direction of increasing ϕ_0. It appears that along such curves the values of ν_0 change monotonically. For small values of ν_0 we obtain points at the right-hand lower part of the curve corresponding to smaller values of ϕ_0. With increase in ν_0 we obtain points described to the left and higher, and corresponding to larger values of the angle ϕ_0. As in cases of practical interest ν_0 should not be less than unity, the point on curve ϕ_k = const. corresponding to $\nu_0 = 1$ is the left-hand limit of this curve. On Fig. 6–8 are given broken lines which connect points with $\nu_0 = 1$ or what amounts to the same thing, which connects the left boundaries of curves ϕ_k = const. It is easy to see that the curves ϕ_k = const. can lie both to the left and to the right of the curve $\nu_0 = 1$.

In Fig. 6–8 are also shown broken curves corresponding to other constant values of ν_0:

$$\nu_0 = 0{\cdot}3;\ 0{\cdot}5;\ 0{\cdot}7$$

It is of interest to note that curves of constant values of ν_0 can, in a whole series of cases, intersect. Thus, from Fig. 6–8 it is easy to see that in the

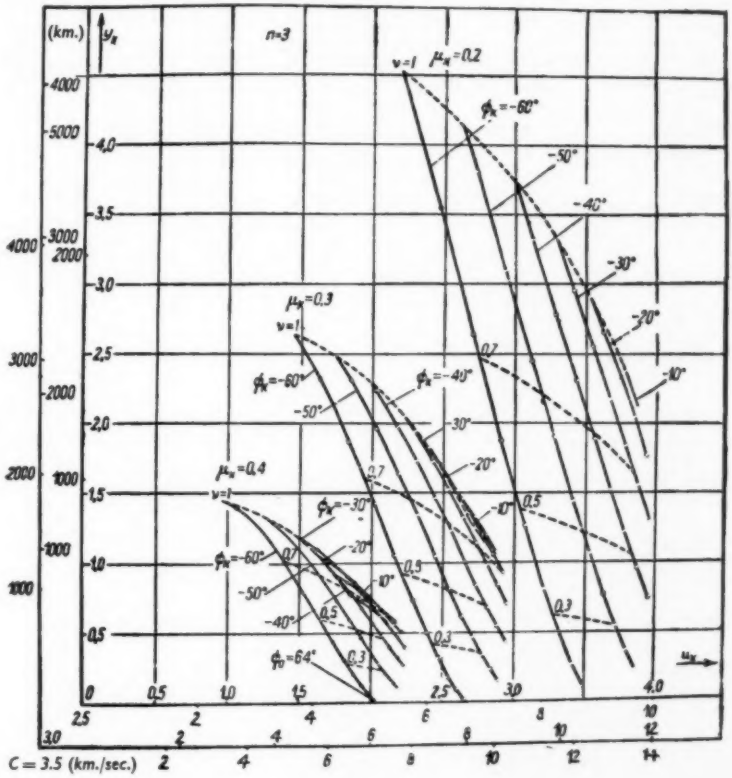


FIG. 7.

cases $n = 2$, $\mu_k = 0{\cdot}3$; $n = 3$, $\mu_k = 0{\cdot}4$; $n = 4$, $\mu_k = 0{\cdot}5$, curve $\nu_0 = 1$ intersects curves $\nu_0 = 0{\cdot}7$, $\nu_0 = 0{\cdot}5$ and apparently with curve $\nu_0 = 0{\cdot}3$ (if the curve $\nu_0 = 1$ is continued in the direction of positive values of ϕ_k). This means that for given values of n and μ_k and for two different values of ν_0 which correspond to given values of ν_0 the optimum programmes can result in obtaining one and the same combination (y_k, u_k).

So as to explain the significance of this effect we will introduce an example. Let us assume that for $n = 2$, $\mu_k = 0{\cdot}3$, $\nu_0 = 0{\cdot}5$ and for the height $\tilde{y}_k$ equals 1·53 (put in non-dimensional form) it is essential to find an optimum programme allowing one to obtain the maximum velocity $\tilde{u}_k$.

Using Fig. 6, by interpolation we find the parameters of the optimum programme ϕ_0 and ϕ_k:

$$\phi_0 \approx 68°, \quad \phi_k \approx -43°.$$

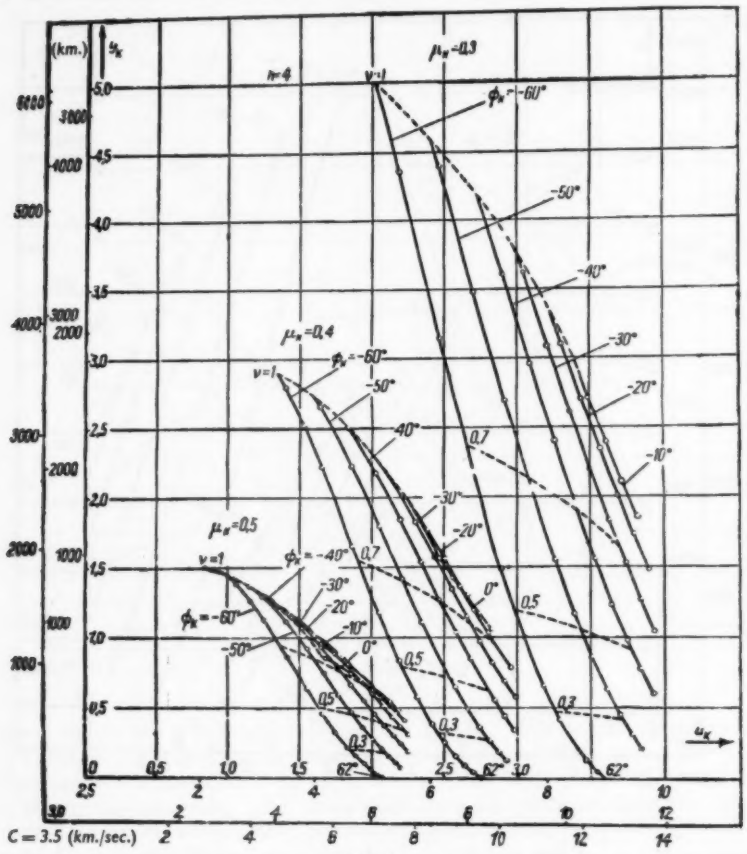


FIG. 8.

This optimum programme allows one to obtain the maximum velocity $\tilde{u}_k$, where $\tilde{u}_k = 0{\cdot}46$ (also taken in non-dimensional form). However, from the same Fig. 6 it is easy to see that the same combination (y_k, u_k) can be obtained by using another value of ν_0, namely $\nu_0 = 1$. In this case the parameters of the optimum programme ϕ_0 and ϕ_k will then be different:

$$\phi_0 = 64°, \quad \phi_k = 0.$$

It must be borne in mind that if the initial mass of the rocket is kept constant, then greater values of ν_0, according to formula (2.18), will correspond with lesser values of rocket thrust P and, therefore, a lesser weight of the engine itself. Then retaining n and μ_k but providing the value $\nu_0 = 1$ instead of

$\nu_0 = 0{\cdot}5$ we have the possibility of installing a bigger payload in the rocket which will not affect the basic flight characteristics of the rocket, y_k and u_k.

From these graphs we see also that for a given number of steps in the rocket, and with a given μ_k there are definite limitations on the possibility of obtaining various combinations of height and projection velocity. These limitations have a dual character. So as to demonstrate this, we will break down all the curves $\phi_k = \text{const.}$ into two classes. Into the first class we put curves having portions which are located to the right and higher than curve $\nu_0 = 1$. In this class we have, for example, all curves $\phi_k = \text{const.}$ corresponding to the case $n = 2$, $\mu_k = 0{\cdot}3$, or, for example, curves $\phi_k = -20°$ and $\phi_k = -10°$, corresponding to the case $n = 2$, $\mu_k = 0{\cdot}2$ (Fig. 6). In the second class we have curves which lie fully to the left and lower than the broken line $\nu_0 = 1$. To this class belong, for example, all curves $\phi_k = \text{const.}$, corresponding to the case $n = 2$, $\mu_k = 0{\cdot}1$, or, for example, curves $\phi_k = -60°$, $-50°$, $-40°$, $-30°$, corresponding to the case $n = 2$, $\mu_k = 0{\cdot}3$ (Fig. 6).

From Fig. 6–8 it is easy to see that for families of the first class of curves it is easy to construct an envelope touching the curves at equal values of μ_k (these curves are not shown). These envelopes set limits in the plane u_k, y_k to some region of practibility and this limitation has the character of energy and is connected with the possibility of obtaining for a given propellent reserve only certain limited heights and velocities.

The other limitation corresponds wholly to the second class of curves and is connected with the practical unfavourable utilization of rockets, in which the thrust of the first stage is less than the initial weight. The broken line $\nu_0 = 1$ will in this case be the limiting curve in the plane (y_k, u_k), above which we will not be able to rise, if n and μ_k are given.

Combining the family of curves of the first class with the limiting family of curves of the second class, we can obtain a general limiting curve connecting the region of attainable combinations of y_k and u_k for given values of n and μ_k. This limiting curve gives us the greatest values of velocity attainable at a given height for given values of n and μ_k, or the greatest values of height for a given final velocity at given values of n and μ_k. In addition, these limiting curves show the greatest values of μ_k at which it is possible from energy considerations to attain given heights and velocities. Points on curves $\mu_k = \text{const.}$ lying on the limiting curve will correspond to values of ν_0 which are the most favourable from the energy point of view.

On Fig. 6–8 the fundamental scale is that for non-dimensional speeds and heights $\tilde{u}_k$ and $\tilde{y}_k$ where

$$\tilde{u}_k = u_k/c, \qquad \tilde{y}_k = y_k/(c^2/g)$$

Because of this, these graphs have a universal application and are suitable for using in cases of different exhaust velocities of the gas from the nozzle of the rocket engine. On these graphs are three dimensional scales, which correspond to three different exhaust velocities:

$$c = 2{\cdot}5 \text{ km./sec.}; \ 3 \text{ km./sec.}; \ 3{\cdot}5 \text{ km./sec.}$$

For the quoted exhaust velocities part of the curves falls in the region of great heights y_k for which direct application of this theory of delivery in a plane-parallel force field is rather difficult. Nevertheless in this case these curves do allow one to make some qualitative estimates of a general character about the effect of choice of parameters of the rocket and its pitch control programme on the widening of the field of attainable velocities and heights.

3. Problem of Delivering to Orbit with Consideration of the Variable Gravitational Field and Rotation of the Earth

In this section we review the more complicated situation concerning delivery into orbit, taking into consideration the variable gravitational field and rotation of the Earth. Here we deal with the relative motion of the rocket in a system of coordinates fixed to the Earth.

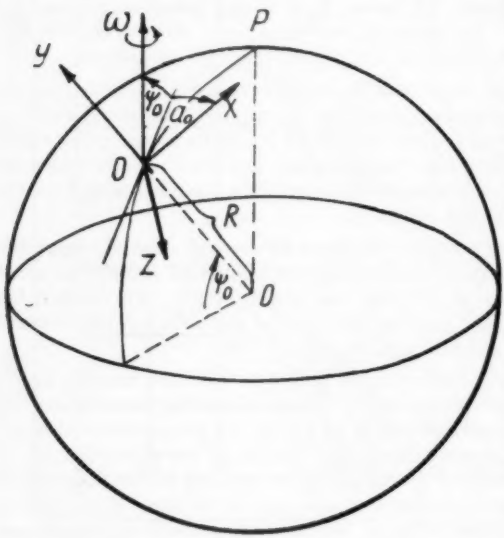


FIG. 9.

Assuming that the trajectory of the rocket with respect to the Earth is plane, we will use the Cartesian system of coordinates, the origin of which is at the starting point, the x-axis directed along the tangent to the Earth in the direction of motion of the rocket, the y-axis directed vertically upwards, the z-axis perpendicular to axes x and y and forms with them a right system of coordinates (Fig. 9). The equations of motion of the rocket in terms of projections on the axes of the Cartesian system of coordinates can be put thus:

$$\ddot{x} = -g_x + p_x - a_{\text{trans}.x} - a_{\text{Cor}.x} \quad .. \quad .. \quad (3.1)$$

$$\ddot{y} = -g_y + p_y - a_{\text{trans}.y} - a_{\text{Cor}.y} \quad .. \quad .. \quad (3.2)$$

$$\ddot{z} = -g_z + p_z - a_{\text{trans}.z} - a_{\text{Cor}.z} \quad .. \quad .. \quad (3.3)$$

where g_x, g_y, g_z are projections of the acceleration due to gravity and p_x, p_y, p_z, are projections of the acceleration due to the reaction force on the axes x, y, z; $a_{\text{trans}.x}$, $a_{\text{trans}.y}$, $a_{\text{trans}.z}$; $a_{\text{Cor}.x}$, $a_{\text{Cor}.y}$, $a_{\text{Cor}.z}$ are respectively the projections of transfer accelerations and Coriolis accelerations on the same axes.

Now let us analyse the individual components of equations (3.1)–(3.3). First of all, it is clear that in view of the condition of rocket motion in the plane x,y, then:

$$\ddot{z} = \dot{z} = z = 0 \qquad .. \quad .. \quad .. \quad .. \qquad (3.4)$$

Furthermore, owing to the same condition, and in view of the central gravitational field, projections of the gravitational acceleration on the axis z should be equal to zero:

$$g_z \equiv 0 \qquad .. \quad .. \quad .. \quad .. \qquad (3.5)$$

Projections of the acceleration of gravity on axes x and y can be put in the form:

$$g_x = \frac{gR^2}{x^2 + (y+R)^2} \cdot \frac{x}{\sqrt{x^2 + (y+R)^2}} \qquad .. \quad .. \qquad (3.6)$$

$$g_y = \frac{gR^2}{x^2 + (y+R)^2} \cdot \frac{y+R}{\sqrt{x^2 + (y+R)^2}} \qquad .. \quad .. \qquad (3.7)$$

Projections of accelerations due to the reaction forces are equal to:

$$p_x = p \cos\phi \cos\beta \qquad .. \quad .. \quad .. \qquad (3.8)$$

$$p_y = p \sin\phi \qquad .. \quad .. \quad .. \quad .. \qquad (3.9)$$

$$p_z = p \cos\phi \sin\beta \qquad .. \quad .. \quad .. \qquad (3.10)$$

where ϕ is the angle between the longitudinal axis of the rocket and plane x,z (pitch angle), β is the angle between the vertical plane intersecting the rocket axis and plane x,y, (angle of yaw), and p, as before, is the modulus of acceleration of the reaction force, a given function of time.

Projections of the Coriolis accelerations $a_{\text{Cor.}x}$, $a_{\text{Cor.}y}$, and $a_{\text{Cor.}z}$ can be easily expressed by kinematic parameters of relative motion, if this formula is used:

$$\mathbf{a}_{\text{Cor.}} = 2[\boldsymbol{\omega} \times \mathbf{v}_{\text{rel.}}] \qquad .. \quad .. \quad .. \qquad (3.11)$$

where $\boldsymbol{\omega}$ is the vector of the angular velocity of rotation of the Earth, and $\mathbf{v}_{\text{rel.}}$ is the relative velocity of the rocket. Notice that for the projection of the angular velocity of rotation $\boldsymbol{\omega}$ on the axes x, y, and z we have the formulae:

$$\omega_x = \omega \cos\psi_0 \cos\alpha_0 \qquad .. \quad .. \quad .. \qquad (3.12)$$

$$\omega_y = \omega \sin\psi_0 \qquad .. \quad .. \quad .. \quad .. \qquad (3.13)$$

$$\omega_z = -\omega \cos\psi_0 \sin\alpha_0 \qquad .. \quad .. \quad .. \qquad (3.14)$$

where ω is the modulus of angular velocity of rotation of the Earth, ψ_0 is the latitude of the firing point, α_0 is the angle between the plane x,y and the meridional plane (the azimuth of the trajectory plane). Using (3.11) for the projection of the Coriolis acceleration, we obtain the well-known formulae:

$$a_{\text{Cor.}x} = 2[\omega \times \mathbf{v}_{\text{rel.}}]_x = \dot{y}\omega_z - \dot{z}\omega_y \quad .. \quad .. \quad (3.15)$$

$$a_{\text{Cor.}y} = 2[\omega \times \mathbf{v}_{\text{rel.}}]_y = \dot{z}\omega_x - \dot{x}\omega_z \quad .. \quad .. \quad (3.16)$$

$$a_{\text{Cor.}z} = 2[\omega \times \mathbf{v}_{\text{rel.}}]_z = \dot{x}\omega_y - \dot{y}\omega_x \quad .. \quad .. \quad (3.17)$$

Finally, notice that because of the conditions of motion $\dot{z} = 0$, the formulae for the projections of the Coriolis component can be represented finally thus:

$$a_{\text{Cor.}x} = \dot{y}\omega_z \quad .. \quad .. \quad .. \quad .. \quad (3.18)$$

$$a_{\text{Cor.}y} = -\dot{x}\omega_z \quad .. \quad .. \quad .. \quad .. \quad (3.19)$$

$$a_{\text{Cor.}z} = \dot{x}\omega_y - \dot{y}\omega_x \quad .. \quad .. \quad .. \quad (3.20)$$

Now we go over to the projections of the transfer acceleration and notice that they are negligibly small as compared with the other terms in equations (3.1)–(3.3), and in solving the problem under consideration, they can be rejected.

Considering these remarks, the system of equations (3.1)–(3.3) can be written in the form:

$$\ddot{x} = -g_x + p\cos\phi\cos\beta - \dot{y}\omega_z \quad .. \quad .. \quad (3.21)$$

$$\ddot{y} = -g_y + p\sin\phi + \dot{x}\omega_z \quad .. \quad .. \quad .. \quad (3.22)$$

$$0 = p\cos\phi\sin\beta - (\dot{x}\omega_y - \dot{y}\omega_x) \quad .. \quad .. \quad (3.23)$$

The last equation of the system (3.21)–(3.23) is used for determining the angle β necessary for compensating side forces and giving the rocket motion in the plane x,y. Consistent with (3.23) we have:

$$\sin\beta = \frac{\dot{x}\omega_y - \dot{y}\omega_x}{p\cos\phi} \quad .. \quad .. \quad .. \quad (3.24)$$

The evaluation of angle β, carried out with the help of formula (3.24), shows that in practical cases of interest angle β will be rather small, and in any case, will never exceed a few degrees, so that when solving this problem under consideration we can put $\cos\beta = 1$ in equation (3.21) without much error.

Assuming that in the problem under consideration the extent of the delivery trajectory is more or less limited, so as to ease the solution of the problem let us simplify the equations for the component of acceleration due to terrestrial gravitation g_x, g_y, replacing formulae (3.6) and (3.7) by corresponding approximate formulae. With this in view we expand the functions $g_x(x,y)$ and $g_y(x,y)$ in the neighbourhood of the point (0,0) in the form of a power series in x and y:

$$g_x = g_x(0,0) + \left(\frac{\partial g_x}{\partial x}\right)_0 . x + \left(\frac{\partial g_y}{\partial y}\right)_0 . y + \ldots, \quad .. \quad (3.25)$$

$$g_y = g_y(0,0) + \left(\frac{\partial g_y}{\partial x}\right)_0 . x + \left(\frac{\partial g_y}{\partial y}\right)_0 . y + \ldots, \quad .. \quad (3.26)$$

where the coefficients of the expansion will be represented by the following formulae:

$$g_x(0,0) = 0, \quad \left(\frac{\partial g_x}{\partial x}\right)_0 = \frac{g}{R}, \quad \left(\frac{\partial g_x}{\partial y}\right)_0 = 0$$

$$g_y(0,0) = g, \quad \left(\frac{\partial g_y}{\partial x}\right)_0 = 0, \quad \left(\frac{\partial g_y}{\partial y}\right)_0 = -\frac{2g}{R}.$$

Furthermore, we will confine the expansion of (3.25) and (3.26) to terms containing x and y to the first power. We will obtain the following approximate formulae:

$$g_x = g.(x/R) \quad .. \quad .. \quad .. \quad (3.27)$$

$$g_y = g - 2g\,(y/R) \quad .. \quad .. \quad .. \quad (3.28)$$

If we take note of all these remarks and simplifications we can put the whole system of equations of motion of the rocket into the following final form:

$$\psi_1 = \dot{u} - 2\omega_z w + \nu^2 x - p\cos\phi = 0 \quad .. \quad .. \quad (3.29)$$

$$\psi_2 = \dot{w} + 2\omega_z u - 2\nu^2 y - p\cos\phi + g = 0 \quad .. \quad (3.30)$$

$$\psi_3 = \dot{x} - u = 0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3.31)$$

$$\psi_4 = \dot{y} - w = 0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3.32)$$

where

$$\nu = \sqrt{g/R} \; .. \quad .. \quad .. \quad .. \quad .. \quad (3.33)$$

$$\omega_z = -\omega\cos\psi_0 \sin\alpha_0 \quad .. \quad .. \quad .. \quad (3.34)$$

As in Section 1, we set up the variational problem to the attainment of maximum velocity at a given height under conditions such that the angle between the final velocity vector and the local horizontal is zero. As before, by reciprocity, the solution obtained will guarantee the attainment of maximum height at a given velocity, and also for given values of height and velocity for a minimum propellent consumption.

Now let us formulate the boundary conditions. Let us assume that at the beginning of motion (at $t = t_0$) we have co-ordinates x_0 and y_0 and some values of horizontal and vertical velocity projections u_0 and w_0. Let us assume moreover that the kinematic parameters x_0, y_0, u_0 and w_0 depend on some parameter, which can be, in particular, the angle θ_0 between the vector of the initial velocity v_0 and the horizontal at the instant $t = t_0$. Introducing into the subsequent formulae this angle θ_0 as such a parameter, we will bear in mind the possibility of replacing θ_0 by some other parameter. Assume that at $t = t_0$:

$$x_0 = x_0(\theta_0), \quad y_0 = y_0(\theta_0), \quad u_0 = u_0(\theta_0), \quad w_0 = w_0(\theta_0) \; .. \quad .. \quad (3.35)$$

At the end of motion at the instant $t = T$ the total height of flight h is fixed and the angle between the velocity vector and local horizontal should be zero. Both these conditions can be written in the following way:

at $t = T$

$$\sqrt{(R + y_k)^2 + x_k^2} - R = h \quad .. \quad .. \quad (3.36)$$

$$\frac{x_k}{R + y_k} = -\frac{w_k}{u_k} \quad .. \quad .. \quad .. \quad (3.37)$$

In the limiting case, if x_k is small, condition (3.37) takes the form:

$$w_k = 0$$

i.e., it is the same as in the problem dealt with in Sections 1 and 2.

Thus we will set out the variational problem. For the given function $p(t)$ we must find a function $\phi(t)$, which will give the maximum horizontal velocity v_k at a given height h, and also the most favourable angle θ_0 between the vector of the initial velocity and the horizontal, corresponding to the starting point. Notice that the relation between the functional problem v_k and the function $\phi(t)$ cannot be put into such a simple form as was possible earlier on.

To solve the problem it is essential first of all to find the first variational function v_k. So as to find the first variation let us use the method of undetermined multipliers due to Lagrange. With this in view and with the initial function:

$$v_k = \sqrt{u_k^2 + w_k^2} \quad .. \quad .. \quad .. \quad (3.38)$$

we will discuss the new function:

$$J = \sqrt{u_k^2 + w_k^2} + \int_0^T H\, dt \quad .. \quad .. \quad .. \quad (3.39)$$

For the function H inside the integral sign we have the expression:

$$H = \lambda_1\psi_1 + \lambda_2\psi_2 + \lambda_3\psi_3 + \lambda_4\psi_4 \; .. \quad .. \quad .. \quad (3.40)$$

where λ_1, λ_2, λ_3 and λ_4 are several, as yet undetermined functions of time, and ψ_1, ψ_2, ψ_3 and ψ_4 are determined in agreement with (3.29)–(3.32). It is clear that the functions (3.39) and (3.38) coincide if conditions (3.29)–(3.32) are fulfilled. The variations of the functions (3.38) and (3.39) will also agree if in varying function ϕ, relations (3.29)–(3.32) will be satisfied, *i.e.*, the following conditions will be fulfilled:

$$\delta\psi_1 = \delta\psi_2 = \delta\psi_3 = \delta\psi_4 = 0.$$

The first variation of function J will take the form:

$$\delta J = \frac{u_k}{\sqrt{u_k^2 + w_k^2}}\,\delta u_k + \frac{w_k}{\sqrt{u_k^2 + w_k^2}}\,\delta w_k + \int_0^T \delta H.dt \quad .. \quad .. \quad (3.41)$$

where $\delta H = H'_u\delta u + H'_{\dot u}\delta\dot u + H'_w\delta w + H'_{\dot w}\delta\dot w + H'_x\delta x + H'_{\dot x}\delta\dot x +$

$$+ H'_y\delta y + H'_{\dot y}\delta\dot y + H'_\phi\delta\phi \; .. \quad .. \quad (3.42)$$

For derivatives H'_u and $H'_{\dot u}$, etc., consistent with (3.29)–(3.32) and (3.40) we obtain the formulae:

$$\left.\begin{aligned} H'_u = 2\omega_z\lambda_2 - \lambda_3;\ \ H'_w = -2\omega_z\lambda_1 - \lambda_4;\ \ H'_x = \nu^2\lambda_1;\ \ H'_y = -2\nu^2\lambda_2; \\ H'_{\dot{u}} = \lambda_1;\ \ H'_{\dot{w}} = \lambda_2;\ \ H'_{\dot{x}} = \lambda_3;\ \ H'_{\dot{y}} = \lambda_4; \\ H'_\phi = p(\lambda_1 \sin\phi - \lambda_2 \cos\phi) \end{aligned}\right\} \quad (3.43)$$

Assuming the functions λ_1, λ_2, λ_3 and λ_4 continuous and differentiable at every point, we integrate by parts the expression within the integral sign on the right-hand side of (3.41), which contains derivatives from the variation. In the end we obtain:

$$\delta J = \frac{u_k}{v_k}\delta u_k + \frac{w_k}{v_k}\delta w_k + \lambda_{1k}\delta u_k + \lambda_{2k}\delta w_k + \lambda_{3k}\delta x_k + \lambda_{4k}\delta y_k -$$

$$-\lambda_{10}\delta u_0 - \lambda_{20}\delta w_0 - \lambda_{30}\delta x_0 - \lambda_{40}\delta y_0 + \int_0^T \delta\tilde{H}dt \quad .. \quad (3.44)$$

where λ_{10}, λ_{20}, λ_{30}, λ_{40} and λ_{1k}, λ_{2k}, λ_{3k}, λ_{4k} are values of functions λ_1, λ_2, λ_3 and λ_4 corresponding to the initial and final instants. Notice that the variations δu_k, δw_k, δx_k and δy_k due to conditions (3.36) and (3.37) are not independent, but are connected mutually by relations:

$$\frac{1}{R+y_k}\delta x_k - \frac{x_k}{(R+y_k)^2}\delta y_k = \frac{w_k}{u_k^2}\delta u_k - \frac{1}{v_k}\delta w_k \quad .. \quad .. \quad (3.45)$$

$$x_k\delta x_k + (R+y_k)\delta y_k = 0 \quad .. \quad .. \quad .. \quad (3.46)$$

Thus from the four variations δu_k, δw_k, δx_k and δy_k only two are independent. From now on we will take functions δu_k and δw_k as the independent variations. Thus in agreement with (3.35) variations δu_0, δw_0, δx_0 and δy_0 are not independent, but are mutually connected by variation $\delta\theta_0$.

Obviously, we have:

$$\delta u_0 = \frac{\partial u_0}{\partial\theta_0}\delta\theta_0,\ \ \delta w_0 = \frac{\partial w_0}{\partial\theta_0}\delta\theta_0,\ \ \delta x_0 = \frac{\partial x_0}{\partial\theta_0}\delta\theta_0,\ \ \delta y_0 = \frac{\partial y_0}{\partial\theta_0}\delta\theta_0 \quad .. \quad (3.47)$$

Using (3.45), 3.46) and (3.47) we will leave out unnecessary variations from the expression standing in front of the integral sign of the right-hand side of (3.44). After simple transformations we then obtain:

$$\delta J = \left\{\frac{u_k}{v_k} + \lambda_{1k} + \frac{w_k}{u_k^2}\left(\frac{R+y_k}{R+h}\right)^2\left[(R+y_k)\lambda_{3k} + x_k\lambda_{4k}\right]\right\}\delta u_k +$$

$$+\left\{\frac{w_k}{v_k} + \lambda_{2k} - \frac{1}{u_k}\left(\frac{R+y_k}{R+h}\right)^2\left[(R+y_k)\lambda_{3k} + x_k\lambda_{4k}\right]\right\}\delta w_k +$$

$$+\left\{\lambda_{10}\frac{\partial u_0}{\partial\theta_0} + \lambda_{20}\frac{\partial w_0}{\partial\theta_0} + \lambda_{30}\frac{\partial x_0}{\partial\theta_0} + \lambda_{40}\frac{\partial y_0}{\partial\theta_0}\right\}\delta\theta_0 + \int_0^T \delta\tilde{H}.dt \quad .. \quad (3.48)$$

Furthermore, for the expression $\delta\tilde{H}$, which stands within the integral in (3.48), we have the formula:

$$\delta\bar{H} = (2w_z\lambda_2 - \lambda_3 - \dot{\lambda}_1)\delta u + (-2w_z\lambda_1 - \lambda_4 - \dot{\lambda}_2)\delta w + (\nu^2\lambda_1 - \dot{\lambda}_3)\delta x + \\ + (-2\nu^2\lambda_2 - \dot{\lambda}_4)\delta y + p(\lambda_1 \sin\phi - \lambda_2 \cos\phi)\delta\phi \quad .. \quad (3.49)$$

Let us now select the multipliers λ_1, λ_2, λ_3 and λ_4 in such a way that in the right-hand side of the expression for variation of the function (3.48) terms containing variations δu, δw, δx, δy appear within the integral sign, and terms containing variations δu_k and δw_k in front of the integral. We then obtain a system of differential equations for λ_1, λ_2, λ_3 and λ_4:

$$\dot{\lambda}_1 = 2w_z\lambda_2 - \lambda_3 \quad .. \quad .. \quad .. \quad (3.50)$$

$$\dot{\lambda}_2 = -2w_z\lambda_1 - \lambda_4 \quad .. \quad .. \quad .. \quad (3.51)$$

$$\dot{\lambda}_3 = \nu^2\lambda_1 \quad .. \quad .. \quad .. \quad .. \quad (3.52)$$

$$\lambda_4 = -2\nu^2\lambda_2 \quad .. \quad .. \quad .. \quad .. \quad (3.53)$$

and also two conditions at the right-hand boundary which can be put in this form:

$$\lambda_{1k} = -\frac{u_k}{v_k} - \frac{w_k}{u_k^2}\left(\frac{R + y_k}{R + h}\right)^2 [(R + y_k)\lambda_{3k} + x_k\lambda_{4k}] \quad .. \quad (3.54)$$

$$\lambda_{2k} = -\frac{w_k}{v_k} + \frac{1}{u_k}\left(\frac{R + y_k}{R + h}\right)^2 [(R + y_k)\lambda_{3k} + x_k\lambda_{4k}] \quad .. \quad (3.55)$$

The first variation of the function (3.39) can be now put in the form directly connecting variations of the function with variations in the angle θ_0 and function $\phi(t)$:

$$\delta J = \left(\lambda_{10}\frac{\partial u_0}{\partial\theta_0} + \lambda_{20}\frac{\partial w_0}{\partial\theta_0} + \lambda_{30}\frac{\partial x_0}{\partial\theta_0} + \lambda_{40}\frac{\partial y_0}{\partial\theta_0}\right)\delta\theta_0 + \\ + \int_0^T p\,(\lambda_1 \sin\phi - \lambda_2 \cos\phi)\,\delta\phi.dt \quad .. \quad (3.56)$$

Assuming that there exists an internal limit to function (3.39), both in θ_0 and in $\phi(t)$ we find it from the condition:

$$\delta J = 0 \quad .. \quad .. \quad .. \quad .. \quad (3.57)$$

As variations $\delta\theta_0$ and $\delta\phi$ are independent, the condition (3.57) will obviously be fulfilled if:

$$\lambda_{10}\frac{\partial u_0}{\partial\theta_0} + \lambda_{20}\frac{\partial w_0}{\partial\theta_0} + \lambda_{30}\frac{\partial x_0}{\partial\theta_0} + \lambda_{40}\frac{\partial y_0}{\partial\theta_0} = 0 \quad .. \quad .. \quad (3.58)$$

$$\tan\phi = \frac{\lambda_2}{\lambda_1} \quad .. \quad .. \quad .. \quad .. \quad (3.59)$$

We have now obtained two conditions, the first of which can be used for finding the optimum angle θ_0, and the second for determining the optimum programme $\phi(t)$.

The system of equations (3.50)–(3.53) and (3.59), together with boundary conditions (3.54), (3.55), (3.58) and also the equations of motion (3.29)–(3.32) and the boundary conditions (3.35)–(3.37), gives a complete solution of this problem. With this relationship the dependence of the optimum control programme on time can be obtained in explicit form.

Actually, system (3.50)–(3.53) is a system of linear differential equations with constant coefficients and can therefore be easily integrated. To obtain a general solution of system (3.50)–(3.53) we must find the roots of the characteristic equation in this system:

$$k^4 - (\nu^2 - 4\omega_z^2)k^2 - 2\nu^4 = 0 \qquad .. \qquad .. \qquad (3.60)$$

Equation (3.60) will have two real and two imaginary roots:

$$k_{1,2} = \pm\nu_1,\ k_{3,4} = \pm i\nu_2 \quad .. \qquad .. \qquad .. \qquad (3.61)$$

where:
$$\nu_1 = \frac{\sqrt{2}}{2}\sqrt{\sqrt{(\nu^2 - 4\omega_z^2)^2 + 8\nu^4} + (\nu^2 - 4\omega_z^2)} \quad .. \qquad (3.62)$$

$$\nu_2 = \frac{\sqrt{2}}{2}\sqrt{\sqrt{(\nu^2 - 4\omega_z^2)^2 + 8\nu^4} - (\nu^2 - 4\omega_z^2)} \quad .. \qquad (3.63)$$

Finding the general solution of system (3.50)–(3.53) and putting it into (3.59), we obtain the general formula for the optimum control programme in pitch, containing the explicit relation between ϕ and time t:

$$\tan\phi = \frac{\sigma_2 \mathrm{ch}\nu_1(T-t) - \sigma_4 \mathrm{sh}\nu_1(T-t) + s_2\cos\nu_2(T-t) - s_4\sin\nu_2(T-t)}{s_1\mathrm{ch}\nu_1(T-t) - s_3\mathrm{sh}\nu_1(T-t) + \sigma_1\cos\nu_2(T-t) - \sigma_3\sin\nu_2(T-t)} \qquad .. \qquad .. \qquad (3.64)$$

where

$$\left.\begin{aligned}
\sigma_1 &= (\nu_1^2 + \nu^2 + 4\omega_z^2)\lambda_{1k} + 2\omega_z\lambda_{4k} \\
\sigma_2 &= (\nu_2^2 + 2\nu^2 - 4\omega_z^2)\lambda_{2k} + 2\omega_z\lambda_{3k} \\
\sigma_3 &= \frac{1}{\nu_2}\left[2\omega_z(\nu_1^2 - \nu^2 + 4\omega_z^2)\lambda_{2k} - (\nu_1^2 + \nu^2 + 4\omega_z^2)\lambda_{3k}\right] \\
\sigma_4 &= -\frac{1}{\nu_1}\left[2\omega_z(\nu_1^2 + \nu^2 - 4\omega_z^2)\lambda_{1k} + (\nu_1^2 + 2\nu^2 - 4\omega_z^2)\lambda_{4k}\right]
\end{aligned}\right\} \qquad (3.65)$$

$$\left.\begin{aligned}
s_1 &= (\nu_2^2 - \nu^2 - 4\omega_z^2)\lambda_{1k} - 2\omega_z\lambda_{4k} \\
s_2 &= (\nu_1^2 - 2\nu^2 + 4\omega_z^2)\lambda_{2k} - 2\omega_z\lambda_{3k} \\
s_3 &= \frac{1}{\nu_1}\left[2\omega_z(\nu_2^2 + \nu^2 - 4\omega_z^2)\lambda_{2k} - (\nu_2^2 - \nu^2 - 4\omega_z^2)\lambda_{3k}\right] \\
s_4 &= -\frac{1}{\nu_2}\left[2\omega_z(\nu_1^2 - \nu^2 + 4\omega_z^2)\lambda_{1k} + (\nu_1^2 - 2\nu^2 + 4\omega_z^2)\lambda_{4k}\right]
\end{aligned}\right\} \qquad (3.66)$$

So as to finalise the problem, we must determine the constants λ_{1k}, λ_{2k}, λ_{3k} and λ_{4k} in formulae (3.65) and (3.66). Let us remember that λ_{1k} and λ_{2k} are

determined from λ_{3k}, λ_{4k} and by means of the kinematic parameters u_k, w_k, x_k and y_k by formulae (3.54) and (3.55).

In this manner formula (3.64) gives the control programme of $\phi(t)$ as a function of time, and also the six parameters λ_{3k}, λ_{4k}, u_k, w_k, x_k, y_k, *i.e.*,

$$\phi = \phi(t; \lambda_{3k}, \lambda_{4k}, u_k, w_k, x_k, y_k) \quad .. \quad .. \quad (3.67)$$

We put $\phi(t)$, determined by formula (3.64), into the equations of motion (3.29)–(3.32). The system (3.29)–(3.32) transforms after this into a non-homogeneous system of linear differential equations with constant coefficients and with variable right-hand sides explicitly dependent upon time. A solution of such a system can be represented in quadratures. As a result of integrating (3.29)–(3.32) we get four solutions connecting the seven parameters u_k, w_k, x_k, y_k, λ_{3k}, λ_{4k} and θ_0. These relations can be written in the form:

$$u_k - u_0(\theta_0) + \Phi_1(\lambda_{3k}, \lambda_{4k}, u_k, w_k, x_k, y_k) = 0 \quad .. \quad (3.68)$$

$$w_k - w_0(\theta_0) + \Phi_2(\lambda_{3k}, \lambda_{4k}, u_k, w_k, x_k, y_k) = 0 \quad .. \quad (3.69)$$

$$x_k - x_0(\theta_0) + \Phi_3(\lambda_{3k}, \lambda_{4k}, u_k, w_k, x_k, y_k) = 0 \quad .. \quad (3.70)$$

$$y_k - y_0(\theta_0) + \Phi_4(\lambda_{3k}, \lambda_{4k}, u_k, w_k, x_k, y_k) = 0 \quad .. \quad (3.71)$$

where Φ_1, Φ_2, Φ_3 and Φ_4 contain integrals, which in the general case are not brought into the elementary functions. Equations (3.68)–(3.71) together with the boundary relations (3.36)–(3.37) and relation (3.58) represent a closed system of seven transcendental equations with seven unknowns—u_k, w_k, x_k, y_k, λ_{3k}, λ_{1k}, θ_0.*

Solving this system we find the value of the kinematic parameters at the end of the trajectory u_k, w_k, x_k, y_k, the optimum value of the initial angle θ_0 and the values of the parameters of the optimum control programme λ_{1k}, λ_{2k}, λ_{3k}, λ_{4k}, where λ_{1k} and λ_{2k} are additionally evaluated with the help of formulae (3.54) and (3.55). The system of equations (3.68)–(3.71), (3.36), (3.37) and (3.58) is quite complicated and unwieldy and to solve it in an explicit form is, in general, impossible. Therefore, for practical use the proposed method of evaluating the optimum programme is to use any numerical integration method for solving this system. We will not stop here at the question of the practical use of the optimum programme found, but we will analyse the formula (3.64) and simplify it.

First of all, we will examine the effect of rotation of the Earth on the selection of an optimum pitch programme and we will show that this influence is negligible.

Let us look at the case when the conditions of motion of the rocket are such that:

$$w_z = 0 \quad .. \quad .. \quad .. \quad .. \quad (3.72)$$

The condition (3.72) is fulfilled, for instance, if $\alpha_0 = 0$ or π, and also if $\psi_0 = \pm\frac{1}{2}\pi$.

* Notice that λ_{10}, λ_{20}, λ_{30} and λ_{40} entering (3.58) are expressed with the help of the final formulae by λ_{3k}, λ_{4k}, u_k, w_k, x_k and y_k.

In this case the formulae (3.62) and (3.63) for ν_1 and ν_2 will be easily simplified and will take the form:

$$\nu_1 = \sqrt{2}\nu, \qquad \nu_2 = \nu.$$

The formulae for σ_1, σ_2, σ_3 and σ_4 can also be simplified and take the form:

$$\left.\begin{aligned} \sigma_1 &= 3\nu^2\lambda_{1k}, \ \sigma_3 = -3\lambda_{3k}, \\ \sigma_2 &= 3\nu^2\lambda_{2k}, \ \sigma_4 = -3\lambda_{4k}/\sqrt{2} \end{aligned}\right\} \quad .. \quad .. \quad (3.73)$$

As far as s_1, s_2, s_3 and s_4 are concerned, in this case from (3.66) we can easily obtain the relation that

$$s_1 = s_2 = s_3 = s_4 = 0 \quad .. \quad .. \quad .. \quad (3.74)$$

Bearing in mind (3.73) and (3.74), for programme $\phi(t)$ we obtain the very much simpler formula:

$$\tan\phi = \frac{\lambda_{2k}\mathrm{ch}\sqrt{2}\nu\,(T-t) + (\lambda_{4k}/\nu\sqrt{2})\mathrm{sh}\sqrt{2}\nu\,(T-t)}{\lambda_{1k}\cos\nu\,(T-t) + (\lambda_{3k}/\nu)\sin\nu\,(T-t)} \quad .. \quad .. \quad (3.75)$$

Formulae (3.75) can be used to a high degree of accuracy as a formula for an optimum programme also in the case when $w_z \neq 0$. This is easily shown if we consider that the rotation of the Earth affects the optimum programme through quantities ν_1 and ν_2.

From (3.62) and (3.63) we see that ω_z enters the formulae for ν_1 and ν_2 only in the combination $(\nu^2 - 4\omega^2_z)$. It is possible to show that $4\omega^2_z$ is small in comparison with ν^2. Using this, with the help of (3.62) and (3.63) we can obtain approximate formulae for ν_1 and ν_2:

$$\nu_1 \approx \sqrt{2}\nu\left[1 - \tfrac{2}{3}\left(\frac{\omega_z}{\nu}\right)^2\right], \quad \nu_2 \approx \nu\left[1 + \tfrac{2}{3}\left(\frac{\omega_z}{\nu}\right)^2\right] \quad .. \quad (3.76)$$

Let us take the maximum possible value of ω_z, equal to ω. It is known that

$$\omega = 7{\cdot}292 \times 10^{-5} \quad .. \quad .. \quad .. \quad (3.77)$$

On the other hand, we evaluate ν by formula (3.33):

$$\nu = 1{\cdot}241 \times 10^{-3} \quad .. \quad .. \quad .. \quad (3.78)$$

Using (3.77), (3.78), we calculate the maximum values of the correcting term in formulae (3.76):

$$\frac{2}{3}\left(\frac{\omega}{\nu}\right)^2 = 0{\cdot}230 \times 10^{-2} \quad .. \quad .. \quad .. \quad (3.79)$$

It can be seen that the correction for ν_1 and ν_2 on account of the rotation of the Earth is really quite negligible, and bearing in mind that this variational problem is solved with a series of simplified formulae for the external forces, it is clear that this correction can really be neglected.

Thus formula (3.75) can indeed be regarded as a general formula for an optimum control programme in pitch, applicable for use in the case of various geographical conditions of delivery into orbit.

Let us now take a look at the limiting form of formula (3.75), corresponding to a very small value of parameter νT. In this case, assuming $\mathrm{ch}\sqrt{2}\nu T$ and $\cos \nu T$ are equal to unity, and $\mathrm{sh}\sqrt{2}\nu T$ and $\sin \nu T$ are equal to νT, in conformity with (3.75) we will obviously obtain:

$$\tan \phi = \frac{\lambda_{2k} + \lambda_{4k}(T - t)}{\lambda_{1k} + \lambda_{3k}(T - t)} \quad .. \quad .. \quad .. \quad (3.80)$$

Formula (3.80) can also be obtained if we solve a given problem without taking into consideration the variation in the field of gravitational force. If in this we consider the terms

$$\frac{x_k}{R}, \quad \frac{y_k}{R}, \quad \frac{T}{\sqrt{R/g}} \quad \text{and} \quad \frac{w_k}{u_k}$$

sufficiently small then we obtain in the end formula (2.1) of the preceding Section, and, with it, also the whole theory of optimum delivery into orbit developed for the plane-parallel gravitational force field.

ERRATA

Heat Transfer in Rockets

The following corrections should be made to the paper by George P. Sutton (*J.B.I.S.*, July–August, 1956, **15** (4), 192–205).

Page 195, Table I, Column 2 (Percentage change in film coefficient: liquid film). *For* 100, 100, 100, 100, 200, 400, 800 and 1,600, *read* 100, 100, 100, 100, 50, 25, 12·5, and 6·25.

Page 202, line 12 from bottom. *For* . . . of the coolant passage, divided by the wetted perimeter, *read* . . . of the coolant passage, where the hydraulic radius is the quotient of the cross-sectional area of the coolant passage divided by the wetted perimeter.

Member urgently requires *Journal of the British Interplanetary Society*, Vol. 10, Nos. 1, 3 and 4. Write: R. Law, 13, The Poplars, Ocean Drive, Ferring-on-Sea, Sussex.

WANTED: *Bulletin of the British Interplanetary Society*, Vol. 2, No. 2, and *Journal of the British Interplanetary Society*, Vol. 6, No. 2. L. S. Snell, 9, Rectory Field, Great Parndon, Harlow, Essex.

Second-hand Books and Periodicals on astronautics bought and sold.—Astronautica, 31, Brooklands Gardens, Hornchurch, Essex.

TECHNICAL REVIEW

Compiled by E. T. B. SMITH, B.Sc., D.C.Ae., A.M.I.Mech.E., A.F.R.Ae.S., FELLOW

Since our last issue, spaceflight has happened. Instead of saying what is going to be done, we can now try and describe what has been done, and things are happening so fast that by the time this article is published it will be very much out of date. However, some of the available data are presented on the Russian vehicles in orbit at the time of writing (early November).

Sputnik I, launched October 4, 1957

The satellite is 184 lb. in mass, with a spherical diameter of 23 inches. It had, when launched, a radio transmitter radiating on 40 and 20 Mc./sec., but this ceased to operate three weeks after launch. The signal consisted of interlaced dash modulations on the two carriers: it is believed that some form of coding was used to give data on ambient temperatures and pressures, and on the Earth's magnetic field. Four whip antennae are fitted. The structure has an aluminium outer shell, and the great mass is probably accounted for almost entirely by batteries, radio, and instruments.

The launching vehicle was apparently made in three stages: the trajectory was initially vertical for about a mile, and it then tilted to 45°. Speed at the end of first stage burnout was 4,500 m.p.h., and at the end of second stage burnout 12,000 m.p.h. After coasting up to orbital altitude, parallel to the Earth's surface, the third stage was fired, giving a final velocity of 18,000 m.p.h. At this instant the nose cone and rocket motor were separated from the satellite: no sighting of either nose cone or the satellite has yet been authenticated, but the rocket is fairly readily visible at dawn and dusk. Initially the satellite led the rocket, but the greater drag on the rocket, combined with its possibly smaller mass, caused it to fall more quickly and thus overtake the satellite. By comparison with the Vanguard launcher, and allowing about 35 lb. for the nose cone, a pessimistic value for the all-up launching weight is about 200,000 lb., or ten times that of the Vanguard launcher. The first stage thrust is thus likely to be about 250,000 to 300,000 lb. wt., probably using three or four engines of 75,000 to 100,000 lb. wt. thrust each. The second stage may have a single engine of the same sort. The propellent combination is believed to be liquid oxygen and alcohol. The third stage has probably a solid propellent rocket motor. A provisional orbit was calculated for October 15, 1957, by the Nautical Almanac Office from the observations made at the Mullard Radio Astronomy Observatory at Cambridge, and details are as given in Table I.

The change in period corresponded to a decrease in height at apogee of 3·5 km./day. Further observations indicated that the satellite was rotating seven times per minute, and that the orbit was precessing (retrograde motion of the ascending node) at $2° 33' \pm 5'$ per day.

The data given above are taken from *Aviation Week*, October 14, 1957, **67** (15), 27–29 and October 21, 1957, **67** (16), 26–31, and from *Nature*,

TABLE I.—*Data *for Sputnik I*

Inclination	64° 40′ ± 10′
Period between successive northward crossings of latitude 52° 12′ (Cambridge)	5750·0 solar sec. ± 0·3 sec.
Rate of decrease of period	2·2 ± 0·1 sec./day
Height at latitude 52° 12′ on northward passage	223 km. ± 10 km.
Height at latitude 52° 12′ on southward passage	470 km. ± 15 km.
Semimajor axis	6937 km.
Eccentricity	0·053 ± 0·001
Latitude of perigee	36° N. ± 3°
Longitude of ascending node	308° 09′ ± 15′
Retrograde motion of ascending node	2° 33′ ± 5′ per day
Meridian passage at Cambridge on October 15	21 08 41 U.T.
Height above mean radius of Earth at perigee	197 ± 10 km.
Height above mean radius of Earth at apogee	934 ± 10 km.

* *Nature*, November 2, 1957, **180**, 879–883.

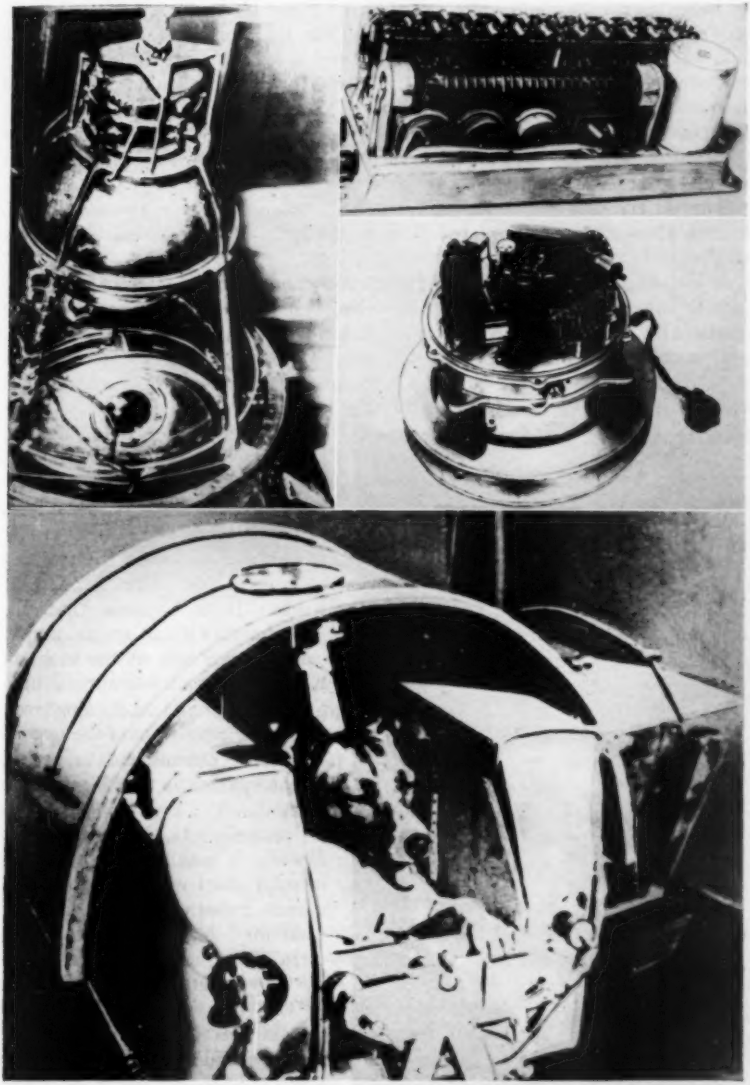
November 2, 1957, **180**, 879–883. The paper in *Nature* was prepared by the staff of the Mullard Radio Astronomy Laboratory, and includes, besides a description of the experimental methods (interferometer and Doppler), preliminary data on the Faraday rotation of the radio signals, the variation of electron density, and the density of the atmosphere. At 200 km. a value of 4×10^{-13} g./cm.^{3}, assuming a temperature of 1,000° K., has been derived for the last-named.

Sputnik II, launched November 2, 1957

This satellite is altogether larger and heavier than Sputnik I, and, as launched, had apogee and perigee distances of 1,050 miles and about 400 miles. The inclination of the orbit was 155°, *i.e.*, a south-easterly launch. The "useful load" was quoted by Prof. Lebedinsky of the Co-ordination Committee of the I.G.Y. as being 508 kg. (1,118 lb.), but it was not certain that this was the entire mass of the satellite or merely the mass of instruments, equipment for maintaining a biologically suitable environment, and the husky bitch passenger, which was used to examine the effects of cosmic rays on a mammalian organism. In any case, a considerable advance over Sputnik I was achieved. The period of revolution was about 6,220 seconds at launch. The satellite was equipped with transmitters in the 20 and 40 Mc./sec. bands, which telemetered information on cosmic rays, the ambient air temperatures and pressures, and the biological condition of the bitch.

If the launching vehicle used conventional propellants (LOX/kerosene) the all-up mass at launch might be of the order of 400–500 tons, but it is reported that the Russians used "new fuels" for the launching, and that these may have been hydrazine, one of the boranes, ammonia, or even liquid hydrogen. Rumours were about that atomic hydrogen had been used.

Pravda for November 13 carried a series of articles on both satellites, together with some photographs (see Fig. 1). Sputnik II consists of a pair of spherical containers, connected by a tubular structure: the larger sphere housed the bitch, with its environmental equipment, and the smaller one had



Ogonëk photographs

FIG. 1. Sputnik II. *Top left:* Arrangement of containers with scientific apparatus. *Top right:* Apparatus for studying cosmic rays. *Centre right:* Apparatus for studying radiation. *Bottom:* Laika in hermetic cabin before mounting on the satellite.

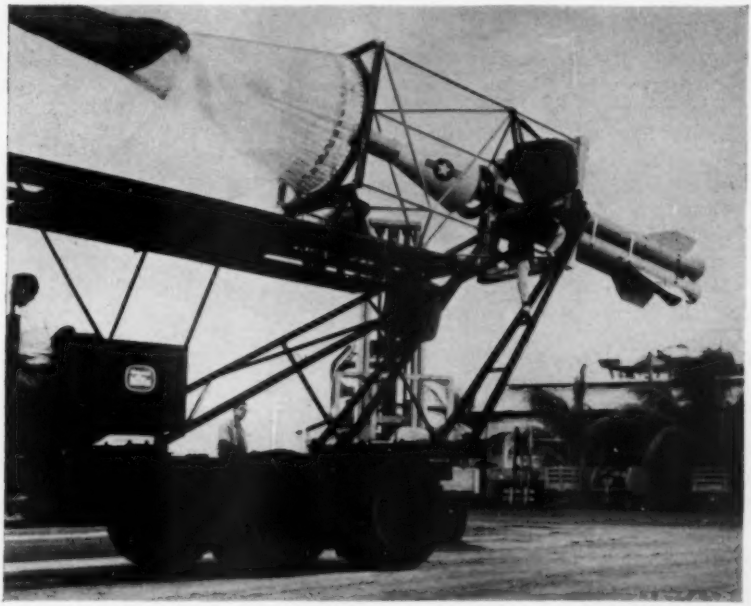
the radio transmitter and solar radiation instrumentation. A nose cone was used to cover the satellite during launch, but this was jettisoned when the orbit was attained: this may be the object seen a few days after the launching. The satellite is, according to *Pravda*, "the last stage of a rocket, housing all the scientific and measuring equipment. This way of housing the equipment appreciably simplified the task of ascertaining the sputnik's bearings by means of optical observation, since, as the experiment with Sputnik I has shown, the tracking of the carrier rocket proved to be far simpler than the tracking of the sputnik itself." This may be so, but none of the pictures so far seen show any details of the rocket to which the two-sphere satellite is attached.

It was believed, when Sputnik II was launched, that an attempt would be made to have the animal re-enter and be recovered. However, *Pravda* carried an article by Prof. V. Parin, of the Medical Academy, which stated "Humanism, an inalienable, salient feature of Soviet science, absolutely rules out the possibility of subjecting to risk the first intrepid conquerors of the Cosmos, who will not be adventure-seekers but the first people of Soviet society . . . For this purpose, science has a tried and tested method, experiments on animals." The English-language *Moscow News* had an article by Prof. V. Cherigovskii, who was involved in the training of animals for rocket flight. He wrote "It is obvious that the experiment with an animal underwent a large risk at the moment when the sputnik was approaching and settling in its orbit. Moreover, under the conditions of the experiment, the animal had to perish, as the sputnik is not to return to Earth."

Official U.S. Air Force Photo

FIG. 2. Project Far Side. The rocket has broken through the balloon and is now 250 ft. above it.

Sputnik II, according to *Pravda*, is making measurements of solar short-wave radiation and cosmic radiation as well as being used for biological studies. The ultra-violet radiations are received by three photomultiplier cells set at 120° to each other: the radiation is selected by optical filters. Battery life was conserved by using a photocell which switched on the measuring circuits only when the Sun's light impinged on one of the photomultipliers.



Official U.S. Air Force Photo

FIG. 3. At Eniwetok Atoll in the Pacific, U.S. Air Force technicians make final checks on the Far Side rocket vehicle and its launch system prior to loosing it beneath its carrier balloon.

The lifetime of Sputnik I was estimated in *Pravda* to be about three months, with a shorter life for the final stage of the launching rocket. No statement was made for Sputnik II except that its lifetime would be "appreciably longer."

This is rather a sketchy description of the Russian satellites, but recourse has had to be made to *The Times* and *The Daily Telegraph* as well as to *Pravda*. No doubt by the time this article is published much more information will have been made available.

Project Far Side

Another attempt at finding out what is going on outside the atmosphere that has reached the operating stage is Project Far Side, which was intended to put a small payload 4,000 miles away from the Earth. The firings were made from Eniwetok Atoll in the Pacific Ocean, and were organized by the U.S.A.F. Office of Scientific Research. A brief description of the vehicle was given in the last TECHNICAL REVIEW, but some photographs are now available which show the system of launching and firing (see Fig. 2–3).

Prof. S. F. Singer of the University of Maryland has written an article on Far Side (*Missiles and Rockets*, October, 1957, **2** (10), 120–122, 124–128), which, as well as describing the vehicle, goes on to discuss phenomena for which Far

Side is better adapted to measure than even Sputnik II, since its intended maximum altitude is higher. The region of the outer ionosphere (250 to 40,000 miles) is one in which the gravitational and magnetic fields of the Earth control the phenomena: beyond this we may consider interplanetary space proper to begin. In the outer ionosphere, there are 600–1,000 ion pairs per cubic centimetre, as well as a large but as yet unknown number of neutral atoms, in which the content of hydrogen is thought to increase with distance from the Earth. Far Side should be able to measure winds, temperatures, and gas composition in this region. The Earth's magnetic field sets up currents in the high ionosphere, which are responsible for magnetic storms and which modify the effect of the Earth's field itself. The particles of cosmic rays are deflected by the magnetic field, and, above the equator, are turned away. In fact, there may be low-energy cosmic rays which never reach the Earth, even near the poles, and Far Side should be able to determine whether they exist at all. If they do, there may be an increase in cosmic ray intensity beyond three or four Earth radii, and this would be bad for interplanetary travellers. The auroral particles (probably high speed protons) are greatly affected by the magnetic field: they may lie closer to the Earth towards the poles than over the equator, but, given sufficient altitude, Far Side should find them, as well as making measurements of the density of dust particles.

A somewhat sad comment on Prof. Singer's paper must be made: six Far Side vehicles were scheduled, but, according to *Aviation Week*, October 28, **67** (17), 31, four had been fired with no success: it was stated that the firings had had the following results:

No. 1, September 25.—Balloon ripped, vehicle not launched.

No. 2, October 3.—Passage of the vehicle through the balloon damaged the telemetry transmitter. Data recorded up to 500 miles.

No. 3, October 7.—Fired from below 70,000 ft., due to cold weather conditions in the stratosphere. Some stages did not fire. Transmitter failed at 400 miles by overheating.

No. 4, October 11.—Balloon froze in tropopause. Rocket fired to destroy it.

No. 5, October 20.—Apparently reached 4,000 miles, based on visual observations of burning period, but transmitter intermittent.

No. 6, October 22.—Also perhaps reached 4,000 miles, but transmitter failed at 1,000 miles.

Radar is not used on Far Side because of the high vertical velocity and small size of the payload. What happens next in the programme depends on funds.

Other Vehicles

The much-advertised, long-awaited, Vanguard must now be considered to be just another satellite; current newspaper reports indicate that during December 1957 some test firings will put a six-inch four-pound satellite into an orbit*. *Aviation Week*, August 26, **67** (8), 86, said, however, that it was unlikely that an orbit would be achieved during test firings, the main purpose of which was to prove the launching vehicle. The small satellites were to have Minitrack transmitters powered with solar batteries.

* Unfortunately, these firings were not successful, but further attempts are scheduled.—*Editor.*

Of interest for its implications regarding the Sputnik launching vehicles is a table given in *Missiles and Rockets* (October, 1957, **2** (10), 73) of some alleged particulars of Russian ballistic vehicles. The data of interest are given in Table II.

TABLE II

Examples of Russian Ballistic Missiles

Name	Type	Length, ft.	Diameter, ft.	Range, miles	Thrust, lb. wt.	Notes
T–1	MRBM	50	5·5	400	77,000	Single stage improved A4 (V-2), LOX-kerosene. Total impulse: $4{\cdot}6 \times 10^6$ lb. wt. sec.
T-2	IRBM	100–125	15	1,800	254,000 (1) 77,000 (2)	Two-stage, improved A4/A9. Stage 2 is T-1 rocket. Total impulse: $20{\cdot}4 \times 10^6$ lb. wt. sec. Weight: 75–85 tons.
T-3	ICBM	100–160	?	5,000	?	Two or three stages. May use T-1 or T-2 parts. Weight: 100–150 tons.
T-3A	Satellite launcher	?	?	?	?	Three or four stages. 1 ton payload, polar orbit, 125–1,000 miles altitude. Uses T-2 or T-3 parts.

According to *Aviation Week*, October 28, 1957, **67** (17), 67, the U.S.S.R. is using, for meteorological and other purposes during the I.G.Y., a sounding rocket called "Meteo": this weighs 1 ton, and stands 29 ft. high. It is in effect a three-stage rocket, with the first stage a hollow cluster of six solid propellent rockets. The second-stage sustainer, using nitric acid and kerosene, fires through the middle of the first stage and is lit at the same time as the first stage. After the boost separates, the sustainer burns out at 44 miles altitude: small solid propellent rockets separate the nose cone and increase the height of the trajectory slightly. A parachute streams from the nose cone at separation, but does not become effective until the cone enters denser air (at about 40 miles). Another parachute lowers the rest of the rocket body. No peak altitude is quoted, but it may be recalled that, at the High Altitude and Satellite Rocket Symposium at Cranfield, Professor Boris Petrov distributed a statement which indicated that "containers reach the altitudes up to 200 km." (124 miles): presumably the Meteo rocket was to be used.

Just to show that the Russians are really uninhibited in their thinking on astronautics, a report in *Missiles and Rockets*, August, 1957, **2** (8), 57 indicates that Prof. Tikhov of the Alma-Ata Observatory in Kazakhstan, who is in charge of the Astro-botany Sector (*sic*) of the Kazakh Academy of Sciences, said that unmanned rockets should reach Venus within ten years, sending back pictures of the surface before landing to determine whether there is any

life on the planet. He also said that his observations show with some certainty that Mars has plant life.

A projected reconnaissance satellite called Pied Piper (and nicknamed Big Brother) is in hand at Lockheed Aircraft Corporation (*Aviation Week*, October 14, 1957, 67 (15), 26 and is intended to carry television, photographic cameras, and infra-red or radar scanner systems. The orbit would be between 300 and 1,000 miles altitude, but no satellite mass is given: a manned version, by 1965, is anticipated.

Hardly a vehicle in our normal sense, but certainly a research project worthy of interest, is Project Stratoscope, which is at present photographing the Sun. This is sponsored by the U.S. Office of Naval Research and the U.S.A.F. Office of Scientific Research, and consists so far of a camera and telescope raised to about 80,000 ft. by a polyethylene Skyhook balloon. The telescope is 114 in. long, and weighs 300 lb.: it has a 12-in. aperture f/8 quartz mirror as the primary element, and the secondary element is a Newtonian quartz mirror, mounted on a swinging arm so that it remains in the Sun's image only long enough to give a photographic exposure. This is necessary to prevent the secondary mirror from being burnt, as it would be if the Sun's rays were concentrated on it continuously. If there be distortion of the primary mirror due to thermal effects, the image quality will suffer, but compensation is provided by a 25× "relay lens" which, as well as increasing the effective focal length to 200 ft., is arranged to scan a region in the primary image field so that, out of twenty different film shots taken in a 20-second cycle, one or two will be in nearly perfect focus. This is intended to ensure that at least 5 per cent. of the 4,000 pictures taken will afford useful data. The telescope is fitted with an automatic pointing device to aim it at the Sun, and a parachute for recovery.

It appears that Stratoscope has given better data about the surface turbulence on the Sun than has been obtained with very large Earth-bound telescopes, due to the elimination of 90 per cent. of the Earth's atmosphere from the telescope's view. Turbulence cells of 300 miles diameter are discernible: "hot magnetic storms" have been seen.

This study of the Sun is expected to provide information about thermonuclear reactions, and future developments are stated to include spectoscopic study of planetary atmospheres from manned balloons, during 1958: a 16 in. telescope will be used at 85,000 ft. Other solar investigations may be made with 36 in. telescopes, using film (1961) and television (1963). Perhaps, some time (1970?), manned satellite observatories may be put up with telescopes of 12 in. or more.

Radio Telescopes

Apart from obtaining much new information about the further distances of the universe, British radio telescopes have shown their versatility by successfully tracking Sputniks I and II. There are two main types of these instruments, which are represented by the installations at Cambridge (Cavendish Laboratory Mullard Radio Astronomy Observatory) and at Jodrell Bank (University of Manchester). The first of these employ cylindrical parabolic

mirrors, formed by suspended wires, as described in an article by the Director, Martin Ryle (*Nature*, July 20, 1957, **180**, 4577). The radio star interferometer is designed for a wavelength of 1·7 m. and comprises a fixed E-W aerial 1,450 ft. long and 65 ft. wide, together with a moving aerial, 190 ft. long in the E-W direction and 65 ft. wide, which travels along railway lines 1,000 ft. long in the N-S direction. The parabolic reflector frames can be rotated about the E-W axis. This array will produce an envelope pattern with a width of approximately 25 × 35 min. of arc to half intensity: there will be an interference pattern with a lobe separation of about 8 min. of arc. The equivalent collecting area of the instrument is about 190,000 ft.² The other main instrument is the pencil-beam array for galactic study on a wavelength of 7·9 m. The fixed E-W aerial is 3,200 ft. long, and the moving aerial will travel 1,700 ft. Each aerial has the form of a corner reflector, with an aperture of 40 ft. The width of the reception pattern is about 1° × 1° to half intensity, and the collecting area is about 200,000 ft². The observing technique which is used demands the employment of a large digital computor to synthesize the data into a conventional "image," and this will be done on the EDSAC computor.

The instrument at Jodrell Bank, under the direction of Prof. A. C. B. Lovell, consists of a movable parabolic dish with aerials for either a radio receiver or a radar transmitter and receiver mounted at the focus. Some details of the engineering work (*Mechanical World*, September, 1957, **137**, 3458) show that the bowl diameter is 250 ft. with the aerials mounted on a 62½ ft. tall pylon in the centre. The bowl weighs 800 tons, and is a steel lattice structure covered with 1½ acres of steel sheet: it is mounted in trunnions at the tops of two 180 ft. towers mounted at the ends of a diametral girder, pivoted in the middle and supported at the ends by bogies running on a circular railway track nearly 180 ft. in diameter. The telescope is rotated in azimuth by driving the bogies, and in elevation by racks bearing on pinions in the trunnions. Braked automobile wheels bearing on a circular arc girder under the bowl are used to stabilize the motion in elevation. Positioning of the bowl is accomplished by electric motors under electronic controls, to an accuracy of ± 12 sec. of arc. Motions available are:

tracking a radio source across the sky;
scanning an area for search purposes;
traversing from one position to another at high speed.

Propulsion

A paper by J. F. Tormey of the Rocketdyne Division of the North American Aviation (*Aircraft Engineering*, August, 1957, **29**(342), 248–251) gives a thorough review of the limitations and potentialities of rocket propulsion systems based on purely chemical means, not including the recombination of active radicles and dissociated species. The treatment is conventional, being based on the classical equations for specific impulse and its dependence on combustion chamber temperature and on the mean molecular weight and the ratio of specific heats (if we can use such concepts at high temperatures) of the products.

Two tables are reproduced, which are convenient for reference (see Tables III and IV). They refer to a chamber pressure of 500 lb./in.² absolute, an

exhaust pressure of 14·7 lb./in.^{2} absolute, and frozen equilibrium expansion (*i.e.*, no change in gas composition after passing the nozzle throat).

A. J. Zaehringer has compiled a Solid Propellent Bibliography (*Jet Propulsion*, August, 1957, **27** (8), 900–927, which, like the earlier bibliographies in *Jet Propulsion* on Project Squid and on jet systems, is a valuable compilation of unclassified literature. It covers internal and external ballistics, chemistry, heating, propellent characteristics, and testing, among other topics. Sources from all over the world, in the period 1850–1956, are quoted.

TABLE III.—*Propellent Systems and Their Performance*

Oxidant	Fuel	T_c (° F.)	M	γ	I_{sp}
Hydrogen peroxide	Gasoline	4830	21	1·20	248
Hydrogen peroxide	Hydrazine	4690	19	1·22	262
Nitric acid	Gasoline	5150	25	1·23	240
Nitric acid	Aniline	5100	—	—	235
Nitric acid	Ammonia	4220	21	1·24	237
Oxygen	Alcohol	5560	22	1·22	259
Oxygen	Gasoline	5770	22	1·24	264
Oxygen	Hydrazine	5370	18	1·25	280
Oxygen	Hydrogen	4500	9·0	1·26	364
Fluorine	Ammonia	7224	19	1·33	306
Fluorine	Hydrazine	7940	19	1·33	316
Fluorine	Hydrogen	5100	8·9	1·33	373

TABLE IV.—*The Bounding Parameters of Chemical Propellent Systems*

Highest temperature system: Cyanogen (C_2N_2) + ozone (O_3)	$T_c = 9467°$ F.	$I_{sp} = 270$ (estimated)
Lowest molecular weight system: Hydrogen (H_2) + fluorine (F_2)	$M_w = 8·9$	$I_{sp} = 373$
Lowest C_p/C_v system: Alcohol (C_2H_5OH) + hydrogen peroxide (H_2O_2)	$\gamma = 1·20$	$I_{sp} = 240$
Highest performance system: $F_2 + H_2$	$I_{sp} = 373$	$I_{sp} = 373$

Radio Techniques

Maser amplifiers and oscillators are discussed in two articles in *Aviation Week* (August 19, 1957, **67** (7), 76–7, 81–2, 87, 89 and August 26, 1957, **67** (8), 91–2, 96, 99, 101, 104 by R. W. Damon of the General Electric Co.'s Research Laboratory. The noise level of Masers is of the order of one-thousandth of that of conventional klystrons or travelling wave tubes, due to the absence of electron flow, itself a cause of noise: the Maser uses energy stored in uncharged atoms. Although small and light, with wide bandwidths and a capability of operating at frequencies between 100 and 100,000 Mc./sec., Masers suffer as yet from disadvantages, including the necessity to operate at liquid helium temperatures, low power output (a fraction of a watt) and the need for the rest of the circuitry to have extremely low noise levels also. The articles describe,

by a mechanical analogy, the way in which the gyrating electrons in the paramagnetic salt of the Maser are used to provide amplification, or oscillation at a required frequency. They point out that, for pulse operation, pulse repetition rates up to about 10^6/sec., gains of 10 dB. for a 10 Mc./sec. bandwidth (or 20 dB. for 1 Mc./sec. bandwidth), and noise levels of 0·06 dB., are obtainable. Continuous wave operation requires the use of a different type of Maser, which, instead of having only two energy levels, has three levels: this uses crystals of hydrated gadolinium ethyl sulphate or potassium chromium cyanide.

Another amplifier using atomic processes rather than electronic ones is mentioned. This is a ferromagnetic device, employing garnet or ferrites. In these materials the electron spins are more tightly coupled than in the paramagnetic salts used in Masers, and the devices act somewhat differently. Neither three-level nor two-level Maser-type amplifiers can be made, due to differences in atomic structure and extremely short relaxation times, but the function of the device depends on a non-linearity in the relation between radio-frequency magnetic field strength and the precession angle of the gyrating electrons, so that energy becomes stored in the crystal at sufficiently high field strengths. This stored energy is released, as in the Maser, to produce oscillation or amplification at a signal frequency different from the local driving frequency. These ferromagnetic devices will operate at room temperature, and, with garnet crystals, the noise level is about 3 dB., falling to 0·06 dB. at liquid helium temperature. The driving power is low, and bandwidths are large.

CORRESPONDENCE

(In these letters to the Editor, some passages have necessarily been omitted).

War against the Moon?

SIR,

Now that the age of interplanetary flight has begun we should be thinking of our moon in terms of a base for further exploration, and in particular as a most suitable planet on which to establish future astronomical observatories. It is, therefore, most important that projects involving the explosion of atomic and hydrogen weapons on the Moon be discouraged by our Society.

As there is no atmosphere worth talking about in this connection, an atomic explosion would lift the highly radioactive dust resulting from it and deposit it more or less evenly over a large area of the Moon's surface. A sufficiently big explosion would be capable of depositing the dust over the whole surface of the Moon, thus effectually barring our natural satellite from future human development.

Yours faithfully,

M. W. HARTFORD.

Latchmoor Cottage, Brockenhurst, Hampshire.

November 8.

(There can be few who will disagree with Mr. Hartford's sentiments. The projects he mentions have been suggested by both Russian and American scientists (e.g., *by Professor S. F. Singer in a paper presented to the Eighth International Astronautical Congress), and* Aviation Week *has reported that an attempt to land an atomic warhead on the Moon will take place later this year.–Ed.)*

The Calculation of Fuel Distribution in Step-Rockets

SIR,

I found Professor Williams' paper[1] on the calculation of fuel distribution in step-rockets very interesting. I agree with him that the notation M_0 instead of m_p would be more consistent, and I gladly accept this emendation.

I myself tried to break down the structural mass of a rocket into components which have a different influence on the total mass but I encountered so many uncertainties that I gave it up. Professor Williams' method seems promising, but I would like to point out that the dependency of the structural ratio on the mass ratio, owing to the acceleration, does not enter into his calculations.

As is well known, there is a simple relation between the mass ratio and the acceleration of a rocket. For the maximum acceleration, which interests us here, we have:

$$a_{\max} = \frac{c\,(r-1)}{T}$$

where T is the burning time of a step.

Following the lines of Professor Williams' paper, I introduced this dependency into his basic equations, but the resulting formulae turned out to be too complicated to be of practical use for preliminary calculations.

Yours truly,

M. VERTREGT.

Mozartlaan 73, The Hague, Netherlands.
March 25.

REFERENCE

(1) M. L. Williams, *J.B.I.S.*, 1957, **16**, 211.

The Equation of Rocket Motion

SIR,

In a recent paper[1] in this *Journal* the author derived the equation of rocket motion from fundamental principles. In the concluding section of that paper, it was endeavoured to demonstrate that:

$$c\ln\frac{m_0}{m_1} > lq\,\frac{m_0-m_1}{m_0 m_1}, \quad \frac{m_0}{m_1} > 1,$$

where all symbols are defined in Ref. 1. The demonstration was based, in the end, on inequality (2.15) of Ref. 1. However, inequality (2.15) holds for values of m_0/m_1 only slightly larger than unity. The range of validity can be extended

to $(m_0/m_1) < 5$ by omitting the last step in inequality (2.14). In any event, the demonstration is not sufficient for all values of m_0/m_1 in excess of unity, and hence cannot be regarded as satisfactory. It is believed that the difficulty stems from the weakness of inequality (2.13).

For the case of exponential mass flow:

$$m = m_0 \exp\left(\frac{t}{t_1} \ln \frac{m_1}{m_0}\right)$$

the burn-out velocity is:

$$\dot{s}_1 = c \ln \frac{m_0}{m_1} - \frac{l}{t_1}\left(\ln \frac{m_0}{m_1}\right)^2$$

For this mass flow the demonstration of Ref. 1 is adequate to show that:

$$c \ln \frac{m_0}{m_1} > \frac{l}{t_1}\left(\ln \frac{m_0}{m_1}\right)^2$$

for all $(m_0/m_1) > 1$.

The following errata in Ref. 1 are noted. In equation (1.13) and Fig. 5, the capital **C** should be a lower case **c**. The name of the senior author of (7) is E. J. McShane.

G. LEITMANN.

Department of Engineering, University of California,
Berkeley 4, California, U.S.A.

November 1.

REFERENCE

(1) G. Leitmann, *J.B.I.S.*, 1957, **16**, 141.

SIR,

Professor Leitmann's conclusion[1] that the burnout velocity of a rocket is dependent on the mass flow rate seems to be based on a solution to his equation (2.3) It should be pointed out that under the assumptions used in the limiting process by which equation (2·5) is obtained, *i.e.*, l = constant, $\dot{m}$ = constant, equation (2.3) reduces to the so-called "classical" equation (2.1), the solution of which was given in (2.2) as the familiar mass-rate-independent expression:

$$\dot{s} = \dot{s}_0 + c \ln \frac{m_0}{m_1}$$

It seems that the anomaly proposed arises from the mathematics rather than from nature, particularly when one considers that such a phenomenon would require that a cannon firing projectiles of equal mass with equal velocity would develop a different recoil velocity, in free space, as the projectile were made longer or otherwise changed in shape.

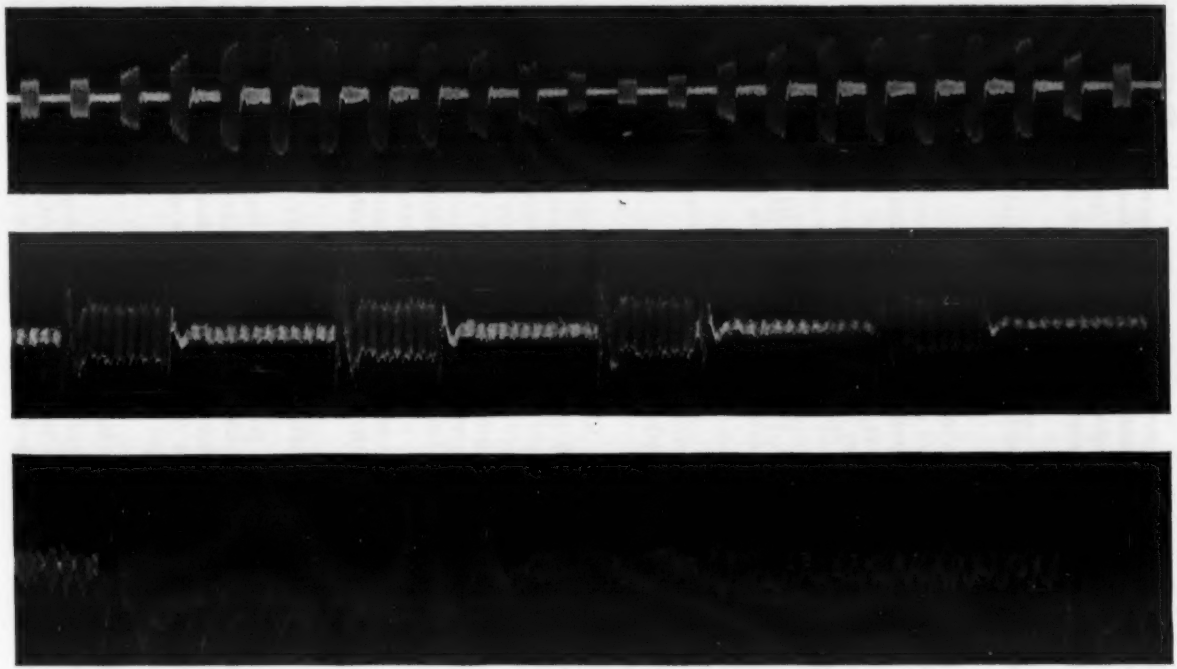
R. LEE AAMODT.

3533 Arizona, Los Alamos, New Mexico, U.S.A.

October 25.

REFERENCE

(1) G. Leitmann, *J.B.I.S.*, 1957, **16**, 141.



TIME——→

FIG. 1–3. Oscillographic records of beeps from Sputnik I.

FIG. 1. *Top:* Compressed time scale (note back wave between beeps). FIG. 2. *Centre:* Ordinary time scale.
FIG. 3. *Bottom:* Single beep shown on extended time scale.

Sputnik I

SIR,

I am enclosing three oscillographic records* of Sputnik beeps as received October 6. Fig. 1 shows the beeps on a compressed time scale to bring out the satellite rotation effect. Fig. 2 shows the frequency modulation in the beeps (vertical stripes), and Fig. 3 gives a detailed representation of a beep on an expanded time scale.

Fig. 4 (a photograph taken by Mr. S. H. Draper of the Dominion Astrophysical Observatory, October 14, 1957) shows the first satellite rocket rising in the northern sky.

Yours truly,

P. E. ARGYLE.

4881, West Saanich Road,
Royal Oak, British Columbia,
Canada.

13 *December*, 1957.

* Reproduced on opposite page.

FIG. 4.

Dominion Astrophysical Observatory, Victoria, B.C., Canada

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B.I.S. NEWS

Nominations for Council; Change in Date of A.G.M.

The Thirteenth Annual General Meeting of the Society is to be held in the Kent Room, Caxton Hall, Caxton Street, London, S.W.1, on **28 June, 1958,** at 6 p.m. The Agenda for the meeting will appear in a later issue of the *Journal*. Members should note that the date fixed for the meeting is different from that previously announced in the Society's printed lecture programme.

In accordance with the provisions of Article 15, the following existing Members of the Council of the Society will retire at this meeting:

J. Foley, K. W. Gatland, P. Moore, G. V. E. Thompson.

In accordance with the provisions of Article 18, Dr. N. H. Langton will also retire at this meeting (being the Member of Council co-opted to fill the vacancy caused by the resignation of Mr. R. A. Smith).

These five members offer themselves for re-election. Other nominations of members for election to the Council are also invited; signed nominations should be forwarded to the Secretary as soon as possible, and in any event not later than 17 May, 1958. If the number of nominations exceeds the number of vacancies, election will be by postal ballot.

Visits and Excursions

Various visits to research establishments and works are being organized. Thus last November a party of members visited the Rocket Propulsion Department of the Royal Aircraft Establishment, and there were visits to the De Havilland Propeller Company and the National Physical Laboratory on 22 February and 1 March respectively. The future programme includes trips to D. Napier & Sons Ltd. on 21 March, and to the De Havilland Engine Co. Ltd. on 12 May. Transport from Caxton Hall is provided for those members who require it.

Provisional reference to visits was made in the Society's lecture programme, and when definite arrangements have been made they are announced to members attending the Society's monthly meetings at Caxton Hall, but it is not always possible to provide all members with printed notices of the final arrangements. If anyone wishes to receive an individual notification of any future visits, he or she is asked to send to the Secretary a stamped addressed envelope marked "Visits" in the lower left-hand corner.

B.I.S. Satellite Programme; Minitrack Radio Measurements

Members are reminded that the Society is organizing a satellite observation programme; a detailed announcement has already appeared in *Spaceflight*, January, 1958, **1** (6), 195. It is hoped that as many members as possible will take an active part in this work, which comes under three categories: Visual Observations, Photographic Observations, and Radio Measurements. Now that both America and Russia have satellites orbiting the Earth and intend to launch others in the immediate future, there is a rapidly increasing need for

observers. It is also possible that in a few months' time an attempt will be be made to fire an unmanned rocket at the Moon; in this type of project individual observations might again play an important part but it is necessary for observers to be prepared, as it is unlikely that any announcement would be made until the rocket was actually on its journey.

If you wish to play a part in the Society's satellite programme, please write either to Headquarters in London, or to your nearest Branch Secretary, giving details of your experience and equipment (if any). You will be sent further details of the Society's programme, together with ephemerides, satellite observation notes, etc.

Among the papers to appear in future issues of the *Journal* is "Project Vanguard Report No. 21: Minitrack Report No. 2: The Mark II Minitrack System," by Roger L. Easton of the Tracking and Guidance Branch, U.S. Naval Research Laboratory. This paper gives all the information necessary for the amateur to construct and calibrate a Mark II Minitrack System for carrying out precision radio observations of U.S. satellites. So that this information can be made available to interested members as soon as possible, a limited number of preprints of the paper are being run off and will shortly be available to members free of charge. To obtain a copy of the preprint, send an envelope measuring at least $6\frac{1}{2} \times 9\frac{1}{2}$ in. to the Society's headquarters. The envelope should be addressed to yourself, bear stamps to the value of $4\frac{1}{2}$d., and the words "Minitrack Reprint" in the lower left-hand corner. No other communication is necessary; the reprints will be despatched as soon as they are received from the printer.

B.I.S. Tie

Arrangements have now been made to provide members with ties bearing the Society's insignia (a rocket and three stars; the lettering which appears on the Society's badge has not been included). This has been somewhat reduced in size and is worked in silver on a celestial blue background. The design is discreet and unobtrusive, yet distinctive enough to be recognizable by fellow-members. The ties are obtainable, price 15s. post free, from the Interplanetary Publishing Co. Ltd., 12, Bessborough Gardens, London, S.W.1.

Election of Members

The following elections were made at the Council Meeting of 2 November, 1957:

Fellows.

Donald Raymond Brincka, B.S., 22630, Clarendon, Woodland Hills, Calif., U.S.A.
William James Bootle, B.Sc., 1917, East Kellogg, Apt. 2, Wichita, Kansas, U.S.A.
Rene Bull, A.M.I.E.E., 51, Lacrosse Avenue, Werneth, Oldham, Lancashire.
Martin Frederick Richard Burton, 19, Four Oaks Gate, Toronto, 6, Ontario, Canada.
John Raymond Carding, B.Sc., 12, Manor Road, Cuddington, Northwich, Cheshire.
Raymond Davies, B.Sc., 10, Earlsway, Curzon Park, Chester.
John Vincent Dobson, A.R.I.C., 31, St. Oswalds Street, West Hartlepool, County Durham.
Peter Edwin Field, 41, Tattersall Gardens, Leigh-on-Sea, Essex.
Kenneth Charles Garner, B.Sc., 12, Prince Philip Avenue, Wharley End, Cranfield, Bletchley, Buckinghamshire.
Alan William Golding, 79, Central Avenue, Gravesend, Kent.
Professor U. Aung Hla, Department of Mathematics, University of Rangoon, Rangoon, Burma.

Richard George Jobbins, 6, Royal York Crescent, Clifton, Bristol, 8.
Walter John Russell King, 6, Cottage Farm Way, Thorpe, Nr. Egham, Surrey.
Frederick Bryan Monge, Ivy Cottage, Holmeton, Nr. Withernsea, E. Yorkshire.
John Charles Balmayne Nunneley, Lindley House, Whitefield Road, New Milton, Hampshire.
Robin Paul Shakespeare, 45, Selsey Avenue, Gosport, Hampshire.
Charles Myron Shure, B.Sc., 64, Rowena Road, Newton Centre 59, Mass., U.S.A.
Robert Colin Skyte, 426, Coventry Road, Hinckley, Leicestershire.
Arthur Norman Smith, 89, Vaux Crescent, Hersham, Walton-on-Thames, Surrey.
Robert Joseph Sunderland, B.Sc., M.A., Ph.D., 6054, N. Vecino Drive, Glendora, Calif., U.S.A.
Arthur Bartlett Whitehead, B.Sc., 12, Heath Green Road, Winson Green, Birmingham, 18.
John Basil Fowler Williams, B.Sc., The Gables, West Kirby, Wirral, Cheshire.

Members

William Henry Richard Anniss, 21a, High Street, Carshalton, Surrey.
Paul Banwell, 65, Symington Road, Fishponds, Bristol.
Leonard Robert Hall Beaumont, B.A., P.O. Box 3127, Auckland, New Zealand.
Arthur Charles Bellotti, 39, Queen Elizabeth Walk, Stoke Newington, London, N.16.
Ronald David Benfield, 11, Windmill Way, Tring, Hertfordshire.
Enrique Beraha, Cerrada Eugenia 27, Mexico 12, D.F. Mexico.
Bushnell Best, B.S., P.O. Box 116, Ransomville, New York, U.S.A.
Zaglul Majid Bishrey, 46T3/21, Karradah, Baghdad, Iraq.
Paul Slayton Boyer, 4, Woodcroft Road, Summit, New Jersey, U.S.A.
Alexander Moir Boyne, 8, Joppa Park, Joppa, Portobello, Midlothian.
James Bramhill, 8, Honeycroft Hill, Uxbridge, Middlesex.
Colin Brown, St. Walstans, London Road, Ryarsh, Nr. Maidstone, Kent.
Douglas Vernon Brown, 14, Rectory Gardens, Doncaster, Yorkshire.
Roy Cameron Christie, LL.B., P.O. Box 993, Wellington, New Zealand.
Colin Victor Clarke, 11, Warwick Avenue, Bedford.
Donald Clarke, 11, Cedar Grove, Haydock, Nr. St. Helens, Lancashire.
Virginia Anne Conolly, Britwell Lodge, Lower Britwell Road, Burnham, Buckinghamshire.
Henry James Cooper, Yame, Cobham, Surrey.
Harvey McLeod Cross, 114, Moss Lane, Hale, Nr. Altrincham, Cheshire.
Robert Charles Culverwell, 27, Powell Street, London, E.C.1.
David Percival Davies, 175, Redland Road, Redland, Bristol, 6.
Kenneth Cecil Day, 7, Newnham Street, Ely, Cambridgeshire.
Nevinson Russell De Courcy, M.A., c/o Australia and New Zealand Bank, 71, Cornhill, London, E.C.3.
Jerry Michael Devine, 434, Heather Street, Victoria, British Columbia, Canada.
Myles Richard Eckersley, 38, Tadfield Road, Romsey, Hampshire.
Audrey Joyce Farnol, 42, Frederick Road, Edgbaston, Birmingham, 15.
David Anthony Farrell, 30, Lady Frances Crescent, Cleethorpes, Lincolnshire.
George William Gee, 9, Pelham Crescent, Beeston, Nottingham.
Maurice William Gregory, 36, Newark Road, Gloucester.
Brian Reginald Grimshaw, 2, Mile Lane, Whipton, Exeter, Devon.
Panaghis John Hadoulis, Down Place Cottage, Nr. Windsor, Berkshire.
Eric Edward Harris, 117, Wensleydale Avenue, Ilford, Essex.
Donald Jackson, 402, Liverpool Road, Birkdale, Southport, Lancashire.
Roy Raymond Kelly, P.O. Box 1, Cholo, Nyasaland.
Brian Douglas Knott, 8, Command Library, GHQ MELF, B.F.P.O. 53.
Thomas Joseph Lardner, 745, President Street, Brooklyn 15, New York, U.S.A.
Sylvanus Lloyd Maunder, Barnsfield, Blundells Road, Tiverton, Devon.
Derek Stewart McAndrew, 53, Latymer Court, London, W.6.
James Dewar McLintock, Bramley Lodge, Hanworth Road, Hampton, Middlesex.
Philip Thevenet Meharg, 4130, 26th Street North, St. Petersburg, Florida, U.S.A.
Thomas Albert Melsom, 121, Rutland Drive, Morden, Surrey.
John Theodore Miller, 92, Cromwell Avenue, Highgate, London, W.6.
John Derek Moore, 22, Victoria Road, Southwick, Sussex.
Robert James Moss, 225, Derby Road, Lenton Sands, Nottingham.
Edward Allen Peterson, 121, Bayard Street, Kane, Pennsylvania, U.S.A.
Robert Warren Powers, 249, Ferry Street, Lawrence, Mass., U.S.A.

THOMAS RAWLINSON, 85, Bellgrove Street, Dennistoun, Glasgow, E.1.
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DENNIS BERNARD ROBERTS, 9, Maple House, Lake Avenue, Bury St. Edmunds, Suffolk.
LEON PAUL ROBERTS, 23, Golders Green Crescent, London, N.W.11.
JAMES WILLIAM ROBILLIARD, 72, Lechmere Avenue, Woodford Green, Essex.
JURGEN ROENNEFAHRT, Ulm/Donau, Burgunderweg 10, Germany.
WALTER LAVERNE SANDERS, 872, Jansen Avenue, San Jose, Calif., U.S.A.
CECIL MARTIN SHARP, The Keive, Bracken Lane, Oaklands, Welwyn, Hertfordshire.
GEOFFREY THOMAS SHELLEY, Brookhill, Kingswear, S. Devon.
NEVIL HARVEY SILVERTON, M.B., Ch.B., M.R.C.S., L.R.C.P., F.R.A.S., 238, Lidgett Lane, Leeds, 17, Yorkshire.
RICHARD DWAYNE SMITH, 1365, St. Paul, Denver, 6, Colorado, U.S.A.
ROBERT CHARLES SNYDER, 408, Morrison Avenue, Cuyahoga Falls, Ohio, U.S.A.
JOHN NIGEL STEVENSON, Mill House, Wherwell, Andover, Hampshire.
JAMES TAYLOR, B.Sc., Sanderson Terrace, Auchtertool, Fife, Scotland.
GORDON TREVOR WALKER, Trevon, 77, Ridgehill Avenue, Gleadless, Sheffield, 12.
RICHARD ARTHUR WEBB, Mens Hostel, S.P.C. Ltd., New Berrima, New South Wales, Australia.
ALEXANDER WILLIAM WEINBAUM, 92, Cazenove Road, London, N.16.
HENRY JAMES WICKENDEN, M.A., 46, Cranley Gardens, London, S.W.7.
JULIA VALERIE WILLCOX, 25, Village Road, Finchley, London, N.3.
WILLIAM ALLEN WORTMAN, Malvern, Iowa, U.S.A.
JOSEPH BURDWOOD WOODFORD, 29, Tangham Walk, Basildon, Essex.

The following elections were made at the Council Meeting of 7 December, 1957:

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EDMUND ATKINSON, B.Sc., 47, Trajan Avenue, South Shields, County Durham.
GORDON HORACE BROWN, 23, Oldfield Road, Coventry.
MAURICE JOSEPH GOLDSMITH, B.Sc., 5, Ladywell Court, East Heath Road, London, N.W.3.
HUGO MAROM, 12, Jeymer Avenue, London, N.W.2.
CASSIM MOHAMAD PATEL, B.E., M.Sc., D.I.C., 35, Nottingham Place, London, W.1.
REGINALD HAROLD PLATT, 9, Raine's Mansions, Wapping Lane, London, E.1.
NEVIL HARVEY SILVERTON, 238, Lidgett Lane, Leeds, 17.
FREDERICK FRANCIS WICKS, 14, Rue des Ursulines, Paris 5e, France.

Senior Members.

SHEILA MARGARET MATHER, 9, Swanwick Street, Old Whittington, Chesterfield, Derbyshire.

Members.

CHRISTIAN MARIE VICTOR ARMANDIAS, "Yvoire," One Pin Lane, Farnham Common, Buckinghamshire.
ARTHUR BARKER, c/o Westminster Bank Ltd., Market Place, Beverley, Yorkshire.
FRANK STANLEY BARROW, 61, Mount Park Avenue, South Croydon, Surrey.
ALEX GEORGE BELL, 42, Sydney Cottage Drive, Bridgnorth, Shropshire.
JOHN BRADLEY, 112, Albion Street, Coatbridge, Lanarkshire.
SHIRLE ROBERT BROOMHEAD, 339, Dunstable Road, Luton, Bedfordshire.
JOHN MONTAGUE BROWNE, 4, Wanda Road, Caulfield, S.E.7, Victoria, Australia.
JOHN STANLEY COOPER, B.Sc., Manor House, Wolborough Street, Newton Abbot, S. Devon.
PHILIP ROYAL FORBES DE CARTERET, Pembroke House, Samares, Jersey, Channel Islands.
EUSTACE IVAN DELAY, 17, Orsett Terrace, London, W.2.
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JOHN ROSS MCCARRON, R.N.Z.A.F. Station, Lauthala Bay, Suva, Fiji.
ANNABELLA MACLENNAN, B.Sc., 30, Errwood Road, Manchester, 19.
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ALAN MANWARING, 21, Hilbury Avenue, Blackley, Manchester, 9.
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ANTHONY JOHN CHARLES PROCTER, "Skaife House," Aberthyn, Cowbridge, Glamorgan.

MAURICE GEOFFREY ROBINSON, 22, Upper Montagu Street, Baker Street, London, W.1.
STANLEY EDWARD ROBINSON, 198, Masterman Road, East Ham, London, E.6.
GILBERT ELLIOTT SATTERTHWAITE, 31, St. James Road, Sutton, Surrey.
HANS FEDERICO CARLOS SEEMANN, P.O. Box 469, Cochabamba, Bolivia.
DAVID LOYD TEMKE, 160, Coatham Road, Redcap, Yorkshire.

Deaths

The Editor regrets to announce the deaths of:

CHRISTOPHER RODNEY ARMSTRONG
ROBERT ESNAULT-PELTERIE (Honorary Fellow).

Obituary notices will appear in a future publication of the Society.

OTHER NEWS

Astronautics and Philately

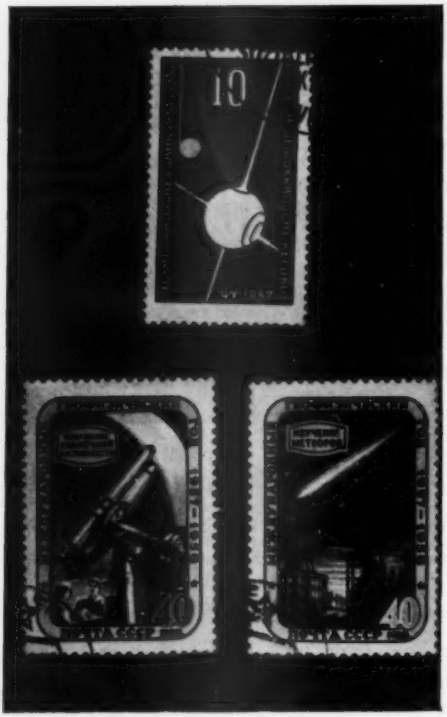

Photo: G. V. E. Thompson

In the development of the rocket between the wars there was a strong connection between astronautics and philately in the attempts (not always unsuccessful) to set up rocket postal services. With the coming of the large guided missile, little or nothing is now being done in this field of use. However, except in the case of British issues, sooner or later practically everything is depicted on a postage stamp, so it was not surprising to find that Sputnik I is shown on a recent stamp issued by the Deutsche Demokratische Republik. This is reproduced approximately same size, at left.

The East German stamp was issued in connection with the International Geophysical Year, and various other countries are also issuing special I.G.Y. stamps. Below, at left, are shown two Russian issues, each for 40 kopecks. One represents the study of solar activity, the other the study of meteors.

As this issue of the *Journal* goes to press, news is received of other I.G.Y. stamps, one of which purports to show Sputnik II in orbit.

The First American Artificial Earth Satellite

U.S Information Service

31 January, 1958: The Jupiter-C blasts off with the Explorer Earth satellite from the launching pad at Cape Canaveral.

JOINT ACTIVITIES

Ninth International Astronautical Congress

Further information is now available concerning this year's International Astronautical Congress, which is to be held in the Netherlands during the week of 25–30 August.

The technical sessions and Council meetings will be held at the University of Amsterdam, situated in the picturesque centre of the town; the programme will also include a reception, banquet, and excursion. Persons wishing to attend the Congress are asked to register by 31 March. The Congress offers an outstanding opportunity of combining a pleasant holiday with attendance at lectures by astronautical experts from many countries. The detailed programme is not yet available, but in an eight-day visit there would probably be time for three or four days' sightseeing in Amsterdam and the surrounding districts. Places of interest in Amsterdam include the many canals, Rembrandt's house, the Rijksmuseum (which has a world-famous collection of old masters, including Rembrandt's "Night Watch"), about forty other museums (modern art, historical, maritime, tropical, etc.), diamond-cutting workshops, and a zoo, and the city is also the home of the celebrated Concertgebouw Orchestra. Places such as The Hague, Rotterdam, Utrecht, Haarlem and the Zuider Zee dam are only a few miles away.

It is hoped that as many members as possible will attend the Congress, and that arrangements can be made for the British delegation to travel as a party, so that a reduction in fares will be obtained. The party would probably fly to Amsterdam on Saturday, 23 August, and return on Sunday, 31 August, but some idea of the number of persons likely to attend is needed before definite bookings can be made. The following information (based on current rates of exchange) may assist members to decide whether they wish to make provisional registration for the Congress.

The registration fee is 60 f. (£5 14s. 0d.), of which 25 f. (£2 7s. 0d.) is payable on submission of the registration form and is non-returnable; the remaining 35 f. is payable on arrival. The registration fee includes the cost of the excursion. The total cost of the flight from London to Amsterdam (including airport charges and road transport to and from the airports) will probably be of the order of £10 to £12 (half fare for children aged 2 to 12). A block booking of hotel accommodation has been made for delegates to the Congress, prices ranging from 14.20 f. to 49 f. (£1 7s. 0d. to £4 14s. 0d.) per day (50 per cent. reduction for children aged up to about six). The hotel charges cover breakfast and one main meal per day, payment for these being obligatory at this season of the year; the prices quoted here include a 15 per cent. service charge.

Members and others who are considering attending the Congress should write at once to the B.I.S. Secretary, who will provide them with further information and the necessary forms for Congress reservation, hotel accommodation, etc. Those who wish to present a paper at the Congress should send *at least two copies* of the manuscript to Dr. L. R. Shepherd, 28, Avon Road, Chilton, Berks.; the closing date for submission of papers is 30 April.

ABSTRACTS

Edited by J. HUMPHRIES and J. FOLEY

A list of abbreviations of titles of journals was included with the 1954 index and addenda have been published in subsequent issues of the *Journal*.

AERODYNAMICS

(217) **Hypersonic studies of the leading edge effect on the flow over a flat plate.** A. G. Hammith and S. M. Bogdonoff. *Jet Propulsion*, **26**, 241–6, 250 (*April*, 1956). Experiments in a variable pressure helium hypersonic tunnel at Mach numbers up to 14. Flow over blunt leading edges shown to be inviscid. (20 *refs.*)

(218) **Laminar heat transfer over blunt-nosed bodies at hypersonic flight speeds.** L. Lees. *Jet Propulsion*, **26**, 259–269, 274 (*April*, 1956). Deals with the limiting cases in which chemical reaction rates are very fast or very slow when compared with diffusion rates across streamlines. (19 *refs.*)

(219) **Aerodynamics at very high altitudes.** S. A. Schaaf. *Jet Propulsion*, **26**, 247–250 (*April*, 1956). A low density supersonic wind tunnel was used to investigate flow conditions in flight at up to eighty miles altitude. (19 *refs.*)

(220) **Methods of calculating the equilibrium laminar heat transfer rate at hypersonic flight speeds.** R. F. Probstein. *Jet Propulsion*, **26**, 497–9 (*June*, 1956). (6 *refs.*)

AIRCRAFT

(221) **The near-constancy of full-power duration for unboosted rocket vehicles** W. P. Berggren. *Jet Propulsion*, **26**, 282 (*April*, 1956). Comment on comparison of full thrust durations of aircraft.

(222) **Generalised theory of the optimum thrust programming for the level flight of a rocket-powered aircraft.** P. Cicala and A. Miele. *Jet Propulsion*, **26**, 443–55, 473 (*June*, 1956). Presents theories for the optimum burning programme for horizontal flight to maximum range. (17 *refs.*)

ASTRONAUTICS

(223) **The spherical differential analyser for solving differential equations arising in space travel problems.** M. J. Bottema. *Space-Flight Problems*, 115–124 (1953). Describes the Hele-Shaw spherical integrator and its use in solving differential equations of interest in space-flight. (6 *refs.*)

(224) **We need a coordinated space program.** W. von Braun. *Space-Flight Problems*, 206–211 (1953). An examination of the problems retarding the achievement of space-flight.

(225) **A general formula for the relation between a field of forces and the orbit of a body therein.** A. L. Cardozo. *Space-Flight Problems*, 103–106 (1953). Demonstrates forms of the functions used in calculating orbits which are suitable for use in differential analysers.

(226) **Programme of action.** G. Crocco. *Space-Flight Problems*, 221–224 (1953). (*In French.*) A general review of the problems of space-flight.

(227) **Refuelling in space and the problem of multi-stage vehicles.** G. A. Crocco. *Space-Flight Problems*, 152–160 (1953). (*In French.*) A scheme for the design of a multi-stage vehicle as a cluster of recoverable tank-rockets surrounding the final stage. The problems of transfer of fuel in space, and of aerodynamic return to the Earth's surface are also discussed. (2 *refs.*)

(228) **Fabrication of the orbital vehicle.** A. E. Dixon, K. W. Gatland and A. M. Kunesch. *Space-Flight Problems*, 125–135 (1953). A detailed description of a vehicle suitable for interplanetary travel between orbits around the terminal planets. It would be constructed in orbit with expendable ferry rockets, each of three steps, crews return to the surface by supersonic glider. (79 *refs.*)

(229) **On the psychology of popular lectures on spaceflight.** E. Dolezal. *Space-Flight Problems*, 218–220 (1953). (*In German.*)

(230) **On plotting small thrust space ship orbits.** J. M. J. Kooy. *Space-Flight Problems*, 107–114 (1953). Assumes that the spaceship has mass very small compared with that of celestial bodies, and hence moves through a potential field known as a function of space and time.

(231) **Co-operation among rocket societies in the present state of astronautical research.** G. Partel. *Space-Flight Problems*, 214–217 (1953). Proposes that the scientific manpower of rocket societies and their output of work should be coordinated internationally.

(232) **A minimum orbital instrumented satellite—now.** S. F. Singer. *Space-Flight Problems*, 136–139 (1953). A proposal for an instrument-carrying satellite vehicle weighing about 100 lb. which could provide worth-while information on solar radiation and geophysics.

(233) **We can have space flight in our time.** R. C. Truax. *Space-Flight Problems*, 212–213 (1953). Discusses the question of financing interplanetary exploration.

(234) **On applications of the satellite vehicle.** R. P. Haviland. *Jet Propulsion*, **26**, 360–63, 368 (*May*, 1956). Lists the uses to which a satellite may be put, and gives data on orbits and observation of the Earth's surface. (11 refs.)

(235) **Satellite ascent mechanics.** J. Jensen. *Jet Propulsion*, **26**, 359 (*May*, 1956). Discusses the guidance accuracy required to establish a satellite in its orbit.

(236) **A history of the artificial satellite.** A. R. Krull. *Jet Propulsion*, **26**, 369–383 (*May*, 1956). A bibliography of approximately 350 references.

(237) **Flight mechanics of ascending satellite vehicles.** F. M. Perkins. *Jet Propulsion*, **26**, 352–58 (*May*, 1956). Describes suitable trajectories, gives methods of computation, and explains methods of optimising the trajectory to reduce propellant consumption. (4 *refs.*)

(238) **Lifetime of satellites in near-circular and elliptic orbits.** N. V. Petersen. *Jet Propulsion*, **26**, 341–351, 368 (*May*, 1956). Gives the equations of motion for spherical and conical body shapes. Shows that lifetimes of satellites increase markedly with altitude and mass-area ratio. (27 *refs.*)

(239) **Control and power supply problems of instrumented satellites.** E. Stuhlinger. *Jet Propulsion*, **26**, 364–68 (*May*, 1956). Describes a device for keeping a satellite aligned with respect to the Earth's closest surface by detecting the direction of minimum cosmic ray intensity. Also reviews different methods of obtaining power for long-duration satellites. (3 *refs.*)

(240) **Instrumenting unmanned satellites.** H. E. Lagow. *Jet Propulsion*, **26**, 496–7 (*June*, 1956). A review of the problems of installation of instruments, and the effect of orbital conditions on collection and transmission of data. (2 *refs.*)

BIOLOGY AND MEDICINE

(241) **Investigations into weightlessness with human and animal subjects in periods of free fall.** H. J. von Beckh. *Space-Flight Problems*, 196–202, (1953). (*In German.*) (22 *refs.*)

(242) **On research methods for the detection of the biological effects of cosmic rays at great heights.** J. Eugster. *Space-Flight Problems*, 203–205, (1953). (*In German.*)

(243) **On the question of orientation under weightless conditions.** S. Gerathewohl. *Space-Flight Problems*, 189–195 (1953). (*In German.*) A discussion of the weightless state as observed by humans and in the Aerobee mouse experiment. (13 *refs.*)

(244) **Hazards associated with 90 per cent hydrogen peroxide aerosols.** C. L. Punte, L. Z. Saunders and E. H. Krackow. *Jet Propulsion*, **26**, 500–1 (*June*, 1956). Reports tests on mice at lethal and sublethal concentrations. (5 *refs.*)

MATERIALS

(245) **Materials for rocket frame and motor construction.** G. P. Casiraghi *Space-Flight Problems*, 163–170 (1953). A review of the materials now available for application to space-vehicle structures, wing surfaces, rocket motors and their linings, and propellent handling systems.

PHYSICS

(246) **The depression of freezing temperatures.** R. M. Corelli. *Space-Flight Problems*, 161–162 (1953). (*In German.*) On the depression of the freezing point of tetranitromethane by the addition of nitrogen peroxide.

(247) **On the problem of a practical thermoelectric current generator for extra-terrestrial applications.** B. Langenecker. *Space-Flight Problems*, 181–188 (1953). (*In German.*) An account of the principles of thermoelectricity and their application.

(248) **Effect of chemical reactions in the boundary layer on convective heat transfer.** D. Altman and H. Wise. *Jet Propulsion*, **26**, 256–58, 269 (*April*, 1956). Analyses the change in heat transfer rate resulting from reactions in the gas phase and on the wall. (6 *refs.*)

(249) **Measurement of total emissivity of porous materials in use for transpiration cooling.** E. R. Eckert, J. P. Hartnett and T. F. Irvine. *Jet Propulsion*, **26**, 280–82 (*April*, 1956). Describes experimental methods and gives results for a porous bronze specimen. (2 *refs.*)

(250) **On laminar boundary layers with heat transfer.** W. D. Hayes. *Jet Propulsion*, **26**, 270–4 (*April*, 1956). Gives a transformation which simplifies the boundary layer equations, and discusses applications of the method. (9 *refs.*)

(251) **Sun-follower for high altitude sounding rocket.** D. D. Terwilliger and G. J. Granros. *Jet Propulsion*, **26**, 275–7 (*April*, 1956). Gives mechanical and electrical data on equipment used for orienting rocket-borne instruments.

(252) **Combustion in turbulent flow.** W. G. Berl *et al. Jet Propulsion*, **26**, 480–495 (*June*, 1956). A collection of short contributions to the Special Panel Meeting held in April, 1955, at the Johns Hopkins University.

(253) **Flame generated turbulence.** W. P. Jensen. *Jet Propulsion*, **26**, 499–500 (*June*, 1956). Warns against comparing measurements obtained in confined and unconfined flames. (3 *refs.*)

(254) **Study of premixed flames using radioactive tracers.** V. Kopytoff, C. G. Bell and A. B. Cambel. *Jet Propulsion*, **26**, 501–2 (*June*, 1956). Describes use and detection of radioactive elements introduced in fuel gas. (7 *refs.*)

PROJECTILES

(255) **The exact solution of the equations of motion of a vertically ascending rocket having its all-burnt point within the atmosphere.** H. G. L. Krause. *Space-Flight Problems*, 171–180 (1953). (*In German.*) Assumes an exponential atmospheric density with height and a mean value for the gravitational acceleration, and allows the drag coefficient to vary with velocity. The A-4 and Veronica rockets are used as illustrations.

(256) **Ballistic missile performance.** J. W. Reece, R. D. Joseph, and D. Shaffer. *Jet Propulsion*, **26**, 251–255 (*April*, 1956). Derives a method for calculating the range of surface-to-surface rockets. (3 *refs.*)

(257) **Ground handling and launching procedures for an alcohol-oxygen rocket missile.** T. M. Pettey. *Jet Propulsion*, **26**, 474–9 (*June*, 1956). Describes handling vehicles and equipment, the launching stand and the operations required to transport, fuel and fire the rocket.

(258) **On the motion of a missile under water.** W. P. Reid. *Jet Propulsion*, **26**, 463–4 (*June*, 1956). Derives the relevant differential equations of motion and produces solutions. (1 *ref.*)

(259) **Optimalising control in the presence of noise interference.** S. Serdengecti. *Jet Propulsion*, **26**, 465–473 (*June*, 1956). Analyses one type of self-adjusting control system, the effect of noise being eliminated by a cross-correlation method. (3 *refs.*)

ROCKET MOTORS

(260) **Some limiting factors of chemical rocket motors.** W. N. Neat. *Space-Flight Problems*, 140–151 (1953). A general review of the factors concerned in the application of rocket motors to interplanetary flight. Concludes that Earth to Moon return journeys are impractical within forseeable development of the chemical rocket. (12 *refs.*)

(261) **Heat-up time of wire glow plugs.** S. K. Clark. *Jet Propulsion*, **26**, 278–9 (*April*, 1956). Gives data on plugs suitable for starting a reaction chamber.

(262) **Comments on "Manifolds for solid propellant rocket motors."** H. Verdier. *Jet Propulsion*, **26**, 279 (*April*, 1956). Communicates experimental results.

REVIEWS

Matériaux et Techniques pour les Engins-Fusées et l'Aviation: Recueil des Conférénces Prononcées lors du Ier Congrès International des Matériaux pour l'Aviation et les Projectiles-Fusées, Paris, Maison de la Chimie, du 28 Mars au Ier Avril 1955. Association pour l'Encouragement à la Recherche Aéronautique A.E.R.A. 27 × 21 cm. Pp. xi + 656 + 241, illustrated. 1956. Paris: A.E.R.A., 1, rue de Courty. (13,000 fr.).

By the use of his grey matter rather than by the exercise of his physical strength, Man long ago gained his predominant position in the living world. He achieved this in part by courage, but also in large measure by gaining knowledge of materials. His rate of acquisition of this vital attribute was, however, like the grinding of the Mills of God, exceeding slow. Initially and for long enough, such learning was derived by trial and error, a technique sanctified by time and not unpractised even today within the orbit of creative designers, stretching from the notably empirical to the refreshingly scientific—say, from architects to aeronautical engineers.

The scientific as opposed to the empirical basis of this type of knowledge is relatively modern. It dates from little over four centuries ago. Then, that universal genius of almost incredible energy, Leonardo da Vinci, conceived the necessity for and carried out tensile tests. Following our Restoration, Newton's critic, Hooke, discovered elasticity. About a century and a half ago, today's materialized and haunting spectre, fatigue, was first seen, as through a glass darkly. Shear came to be appreciated a couple of generations later, while creep was largely discovered by a man now living and by no means yet in the sere and yellow leaf.

So much for a brief résumé of the growth of physical knowledge of the properties of matter in the solid state—one can now rightly ask if this is sufficient for those working in such fields as aviation and guided missiles. The answer is, of course, emphatically no. It is therefore by no means surprising and is indeed entirely fitting that l'Association pour l'Encouragement à la Recherche Aéronautique should, in 1955, have thought it desirable to hold an international conference on materials and techniques. The record of this meeting, now available in this country, is here reviewed.

The work, one can say in at least one sense, is weighty. It turns the scale at 5 pounds 15 ounces. It is of formidable length—some 889 pages without counting advertisements. It is wholly in French. This, unlike that of Chaucer's nun, is not "after the school of Stratford-atte-Bowe," but of the more difficult variety of those who Paris know. It is very hard reading. The pagination is somewhat awkward and there is no index. There are almost ninety different papers, some without references. They range in subjects, in spite of the somewhat restrictive title of the tome, if not from Jericho to June, then less picturesquely, but perhaps with greater metrological accuracy, from thermodynamics to the philosophy of invention, via rocket fuels, combustion, properties of certain metallic materials from 80° to 2500° C. and of some non-metallic ones over a more restricted temperature range, high speed cinematography, airborne accumulators and batteries, etc. and etc.

This gallimaufry is, in fact, a reproduction of papers, without any record of their discussion. It is, as is generally the case with meetings of this type, likely to be of more interest to the participants than to anybody else. No communication of real primary importance seems to have been made, but some good reviews of current knowledge on a number of very varying subjects are certainly given. The writer does not however advise his fellow countrymen to buy this book. It costs over £13, which even in these days of monetary inflation is more than something. It is, however, but seemly to express to the participants the not insincere hope that a good time was had by all, which is about the best one can say to those taking part in a symposium.

P. L. Teed.

Table of Coefficients for Obtaining the Second Derivative without Differences. By Herbert E. Salzer and Peggy T. Robertson. $10\frac{1}{2} \times 8$ in. Pp. 25. 1957. San Diego, Calif.: Convair-Astronautics (Convair Division of General Dynamics Corporation).

These useful tables enable the second derivatives of a tabulated function to be computed at intermediate points without the use of differences. Twofold differentiation of the n-point Lagrange interpolation polynomial leads to the following expression for the second derivative at a point $x = x_0 + ph$:

$$f''\left(x_0 + ph\right) = \frac{1}{h^2 A(n)} \sum_{i=\frac{n-1}{2}}^{i=\frac{n}{2}} A_i^{(n)}(p) f\left(x_0 + ih\right) + R_n(x)$$

where h is the tabulation interval.

The coefficients $A_i^{(n)}(p)$ are tabulated for

$$\begin{aligned}
&(i) \quad n = 5, \ -2 \leqslant p \leqslant 2,\\
&(ii) \quad n = 6, \ -2 \leqslant p \leqslant 3,\\
&(iii) \quad n = 7, \ -3 \leqslant p \leqslant 3,\\
&(iv) \quad n = 8, \ -3 \leqslant p \leqslant 4,\\
&(v) \quad n = 9, \ -4 \leqslant p \leqslant 4.
\end{aligned}$$

Expressions are given to estimate the truncation error $R_n(x)$ and the computation error due to the use of inexact values of $f(x_0 + ih)$.

D. J. CASHMORE.

Irreversible Stochastic Thermodynamics and the Transport Phenomena in a Reacting Plasma. By H. J. Kaeppeler and G. Baumann. (Mitteilungen aus dem Forschungsinstitut für Physik der Strahlantriebe e.v., Stuttgart, No. 8; Final Report, Contract AF GI (514)-939, U.S. Air Research and Development Command.) 30 × 21 cm. Pp. 104 with 6 illustrations. November, 1956. Stuttgart: Verlag Flugtechnik/Ernst von Olnhausen. (DM. 18.—.)

The theory of heat conduction or energy transport in an ionized gas is of considerable value to rocket designers and nuclear fusion physicists as well as to astrophysicists, and this justifies any efforts to develop this difficult subject. A theory must start with calculation of the degree of ionization and its dependence on temperature and pressure. The process of ionization is similar to that of chemical dissociation and we can use the law of mass action. The reaction isochore $K(T, p)$ can be calculated from a knowledge of the partition functions (*e.g.*, Fowler, "Statistical Mechanics," equation 459). The first half of the report is concerned with this calculation. The results calculated for hydrogen at atmospheric pressure show that it becomes ionized rapidly when the temperature rises above 10,000° K., but that at 100 atmospheres the temperatures have to be increased by another 5,000° K., or more, to obtain a similar degree of ionization. This is of course in agreement with Le Chatelier's principle, which requires that a decrease of pressure must favour ionization, because ionization results in an increase in pressure due to the increased number of particles present.

The next task is to calculate the energy transport. Now energy may be transported by radiation, by the electrons, and by the relatively heavy ions and molecules. The radiation transport is ignored, not because it is least, but because it is too difficult to calculate. It is then obvious that if there is appreciable ionization the relatively high mobilities of the electrons will make them much better carriers than the ions. The electron density will be low and hence we shall not get Fermi-Dirac degeneracy as with the electrons in a metal, and we can treat the electrons as ordinary gas particles. The function of the ions will be to make the medium electrically neutral on the average, so that only small local fluctuations from neutrality will cause deviation of the electrons and hence one can apply stochastic methods (probability theory) to determine the effective mean free paths. In addition the fields inside or very close to the ions and molecules will cause capture or scattering of the electrons and the effect of this on the mean free path has to be calculated but is probably less than the effect of the longer-range random scattering. From a knowledge of the effective mean free path or by other methods which are mathematically equivalent, one can calculate the electronic contribution to the energy transport. In addition to the

transport of kinetic energy there may be transport of electron-capture energy or of chemical-recombination energy if the material is not uniformly ionized or dissociated everywhere. This, however, involves ion transport, and will consequently probably be small compared with the transport of kinetic energy by the electrons if the material is at all well ionized. The efforts in the report to extend the calculations in this direction are therefore probably not important. The report concludes by stating that considerable work has still to be done before a satisfactory theory is available.

The reviewer feels disappointed with the report. The main reason is that the authors do not consider any particular model or mechanism for the plasma such as that described above, but hurry through the work hurling all possible pieces of applicable mathematics at the reader and leaving him to see the correlation and relevance. The result is that he has to consult the references before the report becomes intelligible. The report is therefore only valuable to the applied mathematician, but for him it does a very useful job reviewing a most extensive subject. It contains a list of 123 papers or articles on the subject. The reviewer hopes that the incompleteness of the work will encourage the mathematician even though it will disappoint the engineer who wishes to use it.

E. N. ROWLAND.

Solar Eclipses and the Ionosphere. Edited by W. J. G. Benyon and G. M. Brown. $10 \times 7\frac{1}{2}$ in. Pp. x + 330, illustrated. 1956. London: Pergamon Press. (£7).

The Sun is the source of radiations, some of which cannot be observed from ground level. In particular, the far ultra-violet region of the solar spectrum is strongly absorbed by the Earth's atmosphere. It maintains electrically-charged levels in the atmosphere, known collectively as "the ionosphere," which in turn affect the transmission of radio waves and produce a variable component of the Earth's general magnetic field. The Sun is also the seat of disturbances, such as solar flares, sunspots, coronal M-regions, etc., which may give a particle emission as well as abnormal ultra-violet radiations. A whole subject of solar-terrestrial relationships has therefore grown up, in which the astrophysicist co-operates with the geophysicist and the radio physicist.

This volume collects together the texts of some fifty papers which were read at a symposium held in the rooms of the Royal Society in London on 22–24 August, 1955. The papers deal with such subjects as the theory of ion recombination in the ionosphere, solar radio noise, the effects of solar flares on the ionosphere, etc. It is especially valuable to have the discussions that the papers evoked also reported. This is a most important book, for it contains much original work—but it is a book for the specialist, not the general reader. To anyone intimately connected with research into solar physics, the ionosphere and cognate subjects, it is indispensible.

M. W. OVENDEN.

The Making of a Moon. By Arthur C. Clarke. $8\frac{1}{2} \times 5\frac{1}{4}$ in. Pp. 182, with 48 illustrations. 1957. London: Frederick Muller Ltd. (21s.)

The dust cover states that this book "Describes Man's first major step into Space—the building and launching of the Earth Satellites." This statement, together with the book's title, is difficult to reconcile with the contents. At the time of writing, the forthcoming Russian achievement was not apparently anticipated by the author, and his attention was concentrated—in common with that of most people in the Western world—on the American plans. Making the necessary allowance for this, the point of criticism made by this reviewer is that no more than a third of the book is devoted to the Vanguard programme and that only a small fraction of this can be taken as relating to the building and launching of the American satellites.

The book is an interesting commentary on many aspects of the satellite scene. The author deals with the early history of ideas on the subject, the Vanguard programme, the possible scientific, civil and military uses of satellites, and with anticipated future developments. There are sections on the observation of satellites and on legal aspects of satellite flight.

The author's style seems uncharacteristically prosaic when he engages in straight reporting—a task which might better be left to lesser figures. He is at his best when viewing the human aspect of the scene, and the impact of man's activity on the pattern of nature. He modestly makes no reference to his own contributions to the propagation of ideas, though these are by no means negligible.

The book is well printed. The illustrations are excellent and add to the interest of the text. The overall impression left with this reviewer is one of regret that the author did not choose to follow more specifically one path or the other—that of reporting and commenting on the Vanguard programme, or of surveying the satellite field in broader terms.

S. W. GREENWOOD.

Satellite! By Erik Bergaust and William Beller. 8½ × 5½ in. Pp. 239, with 14 illustrations. 1956. New York: Hanover House. ($3.95) 1957. London: Lutterworth Press. (17s. 6d.)

In a foreword, Hermann Oberth describes this book as "highly interesting. And complete!" It is, indeed, a most readable and fascinating work, covering a wide field and dealing on a popular level with almost every conceivable aspect of satellite flight, from their scientific applications to their influence on the future patterns of human society. It deals almost entirely with work carried out or in progress in the United States.

The authors have compiled information from a wide variety of sources, and have added some original interpretation and thought. An unfortunate weakness of the book, in the reviewer's opinion, is the variation in style, which appears to be the result of reproducing the spirit, if not the exact wording, of the particular documents from which material has been abstracted. There are a number of errors, indicating a hurried preparation for publication, though difficult to avoid completely in recording events that are taking place at a rapid rate. There is a regrettable amount of rather untidy expression, in particular the analogy between satellite flight and the behaviour of a mass being rotated at the end of a rubber cord (p. 88). This analogy is so misleading that it needs to be rejected in its entirety. A remark in parentheses on page 223 indicates that the authors do not properly appreciate the need for a jet of high atomic mass in an ion rocket system. There are a number of seemingly dogmatic statements of a debatable nature. The military is described on page 131 as a force whose historical objective has been destruction. There are many readers who will be distracted by this type of over-simplified treatment of an involved matter.

In spite of the above criticisms, the book stands out among contemporary publications as an example of adequate popular treatment of a highly topical subject. One only wishes that its weaknesses had not been present, for then it could have been more widely recommended.

S. W. GREENWOOD.

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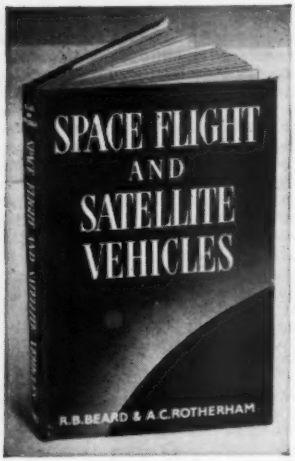

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